USING DERIVATIVE RULES

1. Only the power rule may be used without showing work; must show how terms are rewritten in power rule form before applying the rule.

2. Cannot mix derivative terms and function terms in the same line.

3. Always simplify completely; may not leave a fraction in the numerator or denominator of a fraction.

\[
\frac{3}{4} = \frac{3}{4x^2}
\]

4. No negative exponents are allowed in your answer.

\[5x^{-3} = \frac{5}{x^3}\]

5. Fractional exponents may be left or changed to roots.

\[x^{\frac{2}{3}} \quad \text{or} \quad 3\sqrt{x^2}\]

6. Fractions must use a horizontal bar, except when they are exponents.

\[\frac{2}{3x}, \text{ not } 2/3x\]

\[x^{\frac{2}{3}} \text{ or } x^{\frac{2}{3}}\]
Writing functions in $ax^n$ form -- when roots are involved

To rewrite a root as a fractional exponent, the index of the root (square, cube, fourth, etc.) becomes the denominator of the fractional exponent. The exponent of the variable in the root becomes the numerator of the fractional exponent.

Examples:

$$f(x) = \sqrt[5]{x} = x^{\frac{1}{5}}$$

$$f(x) = 3\sqrt{x} = 3x^{\frac{1}{2}}$$

$$f(x) = \frac{\sqrt[6]{x}}{6} = \frac{1}{6}x^{\frac{1}{4}}$$

$$f(x) = \sqrt[7]{x^5} = x^{\frac{5}{7}}$$

$$f(x) = 4\sqrt[3]{x^2} = 4x^{\frac{2}{3}}$$

$$f(x) = x^3\sqrt{x} = x^3 \cdot x^{\frac{1}{2}} = x^{\frac{7}{2}}$$

Writing functions in $ax^n$ form -- when the variable is in the denominator

$$f(x) = \frac{3}{x} = 3x^{-1}$$

$$f(x) = \frac{1}{9x^7} = \frac{1}{9}x^{-7}$$

$$f(x) = \frac{11}{2x^4} = \frac{11}{2}x^{-4}$$

$$f(x) = \frac{1}{\sqrt[4]{x}} = \frac{1}{x^{\frac{1}{4}}} = x^{-\frac{1}{4}}$$

$$f(x) = \frac{7}{\sqrt[6]{x^5}} = \frac{7}{x^{\frac{5}{6}}} = 7x^{-\frac{5}{6}}$$

$$f(x) = \frac{3}{4\sqrt[4]{x^3}} = \frac{3}{4}x^{-\frac{3}{4}}$$

$$f(x) = \frac{3}{4\sqrt[4]{x^3}} = \frac{3}{4}x^{-\frac{3}{4}}$$
Derivative of a Simple Power Function

A simple power function is a function of the form $f(x) = x^n$, where $n$ is a number. The function may already be written in this form, or it be possible to rewrite it in this form.

The rule for this derivative is: \[
\frac{d}{dx}(x^n) = nx^{n-1} \quad \text{or} \quad \text{if } f(x) = x^n, \text{ then } f'(x) = nx^{n-1}
\]

Examples:

\[
\begin{align*}
f(x) = x & \rightarrow f'(x) = 1x^{1-1} = x^0 = 1, \quad f'(x) = 1 \\
f(x) = x^6 & \rightarrow f'(x) = 6x^{6-1} = 6x^5, \quad f'(x) = 6x^5 \\
f(x) = x^{1.25} & \rightarrow f'(x) = 1.25x^{1.25-1} = 1.25x^{0.25}, \quad f'(x) = 1.25x^{0.25} \\
f(x) = x^{\frac{3}{2}} & \rightarrow f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}x^{\frac{1}{2}}, \quad f'(x) = \frac{3}{2}x^{\frac{1}{2}} \\
f(x) = \frac{1}{x^2} = x^{-2} & \rightarrow f'(x) = -2x^{-2-1} = -2x^{-3} = -\frac{2}{x^3}, \quad f'(x) = -\frac{2}{x^3} \\
f(x) = \sqrt{x} = x^{\frac{1}{2}} & \rightarrow f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}}, \quad f'(x) = \frac{1}{2x^{\frac{1}{2}}} \\
f(x) = \sqrt[4]{x} = x^{\frac{1}{4}} & \rightarrow f'(x) = \frac{1}{4}x^{\frac{1}{4}-1} = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4x^{\frac{3}{4}}}, \quad f'(x) = \frac{1}{4x^{\frac{3}{4}}}
\end{align*}
\]
The Constant Multiple Rule

If a differentiable function is multiplied by a constant $c$, take the derivative of the function and multiply it by $c$. In math notation, \[ \frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x)). \]

We will be using this rule in conjunction with the simple power rule, or \[ \frac{d}{dx}(c \cdot x^n) = c \cdot nx^{n-1} = (c \cdot n)x^{n-1} \quad \text{or, if } f(x) = cx^n, \text{ then } f'(x) = (c \cdot n)x^{n-1} \]

Examples:

- $f(x) = 2x^4 \rightarrow f'(x) = (2 \cdot 4)x^3 = 8x^3 \quad f'(x) = 8x^3$
- $f(x) = \frac{1}{2}x^5 \rightarrow f'(x) = \left(\frac{1}{2} \cdot 5\right)x^4 = \frac{5}{2}x^4 \quad f'(x) = \frac{5}{2}x^4$
- $f(x) = 6x^{\frac{4}{3}} \rightarrow f'(x) = \left(6 \cdot \frac{4}{3}\right)x^{\frac{1}{3}} = 8x^{\frac{1}{3}} \quad f'(x) = 8x^{\frac{1}{3}}$
- $f(x) = 0.5x^{2.2} \rightarrow f'(x) = (0.5 \cdot 2.2)x^{1.2} = 1.1x^{1.2} \quad f'(x) = 1.1x^{1.2}$
- $f(x) = \frac{1}{2x^3} = \frac{1}{2}x^{-3} \rightarrow f'(x) = \left(\frac{1}{2} \cdot -3\right)x^{-4} = -\frac{3}{2}x^{-4} = -\frac{3}{2x^4} \quad f'(x) = -\frac{3}{2x^4}$
- $f(x) = -\frac{3}{x^2} = -3x^{-2} \rightarrow f'(x) = (-3 \cdot -2)x^{-3} = 6x^{-3} = \frac{6}{x^3} \quad f'(x) = \frac{6}{x^3}$
- $f(x) = \frac{2}{3}\sqrt{x} = \frac{2}{3}x^{\frac{1}{2}} = \left(\frac{2}{3} \cdot \frac{1}{2}\right)x^{-\frac{1}{2}} = \frac{1}{3}x^{-\frac{1}{2}} = \frac{1}{3x^{\frac{1}{2}}} \quad f'(x) = \frac{1}{3x^{\frac{1}{2}}}$