- **1.** Limits of functions of several variables:  $\lim_{(x,y)\to(x_0,y_0)} f(x,y)$ , etc.
- **2.** Partial derivatives, mixed partials:  $f_{xxy}$ ,  $\frac{\partial^2 f}{\partial y \partial x}$ ,  $f_{yxy}$ , etc.
- **3.** The Chain Rule; Tree diagrams; Implicit Differentiation for functions defined implicitly by f(x, y) = 0 or f(x, y, z) = 0, etc.
- 4. Gradients  $\nabla f$ , properties of gradients ( $\nabla f \perp$  level curves/surfaces, etc); Rate of change of f along a curve  $C : \mathbf{r}(t)$ ; directional derivative  $D_{\mathbf{u}}f$ .
- 5. Equation of tangent plane to surfaces defined as level surfaces f(x, y, z) = C or surfaces which are graphs of z = f(x, y); equation of normal line to surfaces; linearization L(x, y) of f at a point say  $(x_0, y_0)$ ; estimating changes in f using total differential of f.
- 6. Local/Relative max and min and saddle points of f(x, y); Hessian;  $2^{nd}$  Derivative Test.
- 7. Absolute max and min over closed and bounded (compact) regions.
- 8. Max and min of functions with a constraint; Method of Lagrange Multipliers.
- **9.** Taylor's Formula for functions of several variables (recall **Basic Five Maclaurin Series**); Quadratic and Cubic Approximations.
- **10.** Partial derivatives with a constraint:  $\left(\frac{\partial x}{\partial z}\right)_{t,y}, \left(\frac{\partial w}{\partial t}\right)_{x}$ , etc.
- 11. Double integrals over regions  $D \subset \mathbb{R}^2$ ; Iterated integrals; Fubini's Thm; V-type and H-type regions; Change Order of Integration; areas and volumes using double integrals; the average of f(x, y) over a region D (i.e.,  $f_{av}$ ); Change of Variables Formula in Polar Coordinates.
- 12. Triple integrals over solid regions  $D \subset \mathbb{R}^3$ ; Iterated integrals; Projections of solid region D onto xy, yz, xz- planes; change order of integration; volumes using triple integrals; the average of f(x, y, z) over a solid D (i.e.,  $f_{av}$ ).
- 13.

14.

15.