(1) Approximate the actual solution of \( \begin{cases} y' = x + 2y \\ y(0) = 1 \end{cases} \) at \( x = 0.6 \) using the Euler Method with \( h = 0.2 \). Do this by hand and show all computations.

(2) Find the actual solution \( \phi(x) \) of the initial value problem above and use the Euler Method (eul) with \( h = 0.2 \) to complete this table:

<table>
<thead>
<tr>
<th>( x_n )</th>
<th>Euler Approximation</th>
<th>Actual Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(3) Consider the initial value problem \( y' = -2y + e^{-x}, \quad y(0) = 1. \)

(a) Solve this initial value problem and find \( y(1). \)

\[
y = \quad [\text{box for answer}]
\]

\[
y(1) = \quad [\text{box for answer}]
\]

(b) What is the smallest value of \( n \) for which the Euler Method (eul) with \( n \) steps \( (h = \frac{1}{n}) \) will give a value \( y_n \) that approximates the actual solution at \( x = 1 \) within 0.05?

\[
n = \quad [\text{box for answer}]
\]

(c) Use dfield5 to plot (on the same graph) the solutions of \( y' = -2y + e^{-x} \) satisfying \( y(0) = 0.95, y(0) = 1 \) and \( y(0) = 1.05. \)

(Attach the graph at the end of this worksheet)
Consider the initial value problem \( y' = 2y - 3e^{-x}, \quad y(0) = 1 \).

(a) Solve this initial value problem and find \( y(1) \).

(b) What is the smallest value of \( n \) for which the Euler Method (\texttt{eul}) with \( n \) steps (\( h = \frac{1}{n} \)) will give a value \( y_n \) that approximates the actual solution at \( x = 1 \) within 0.05?

(c) Use \texttt{dfield5} to plot (on the same graph) the solutions of \( y' = 2y - 3e^{-x} \) satisfying \( y(0) = 0.95, \ y(0) = 1 \) and \( y(0) = 1.05 \).

(Attach the graph at the end of this worksheet)

(5) Using the plots in (3)(c) and (4)(c) above, briefly explain why \( n \) is larger in one case rather than the other to obtain the same degree of accuracy.
(6) Give reasons why the Euler Method with $h = 0.1$ does not give a good approximation of the actual solution at $x = 1$ of these initial value problems:

(a) $y' = (y + \frac{5}{4})^2, \quad y(0) = 0 \quad \text{Actual solution: } y = \frac{25x}{4(4 - 5x)}$

(b) $y' = \frac{25x}{32(1 - 2y)}, \quad y(0) = 0 \quad \text{Actual solution: } y = \frac{1}{8}(4 - \sqrt{16 - 25x^2})$