Worksheet # 3
Taylor Series

(1) Consider the initial value problem \( \begin{cases} y' = xy^3 - 9x \\ y(1) = 2 \end{cases} \)

(a) Compute \( y'(1) \). Is the solution to the IVP increasing or decreasing near \( x = 1 \) ?

\[ y'(1) = \quad \text{Solution is } \quad \text{near } x = 1 \]

(b) Compute \( y''(1) \). Is the solution to the IVP concave up or concave down near \( x = 1 \) ?

\[ y''(1) = \quad \text{Solution is } \quad \text{near } x = 1 \]

(2) Find the first three nonzero terms of the Maclaurin Series of the solution of the initial value problem \( y' = xy, \quad y(0) = 1 \).

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(3) Find the first four nonzero terms of the Maclaurin Series of the solution of the initial value problem
\[ \begin{cases} y'' - 2y' + y = 0 \\
y(0) = 0 \\
y'(0) = 1 \end{cases} \]

(4) (a) Find the first four terms of the Taylor Series about \( x_0 = 1 \) of the solution of the initial value problem \( y' - 2y = e^{2x}, \ y(1) = 0. \)

(b) Plot the graphs of the polynomial obtained in (a) and the actual solution \( y = e^{2x}(x - 1) \) over the interval \([0.0, 1.7]\). Attach both graphs to this worksheet.