Chapter 4 - Complex Integration

1. Complex integrals over an interval \( \int_a^b \{u(t) + iv(t)\} \, dt \).

2. Contour integrals \( \int_{\Gamma} f(z) \, dz \); **Fundamental Theorem of Calculus for Contour Integrals**.

3. \( ML \)-Inequality \( \left| \int_{\Gamma} f(z) \, dz \right| \leq ML \).

4. **Cauchy’s Theorem; Generalized Cauchy’s Theorem**.

5. Applications of Cauchy’s Theorem:
   - I. **Cauchy’s Integral Formula**
   - II. **Cauchy’s Integral Formula For Derivatives**
   - III. \( C^\infty \) Theorem
   - IV. **Morera’s Theorem**
   - V. **Maximum-Modulus Principle**
   - VI. **Liouville’s Theorem**
   - VII. **Fundamental Theorem of Algebra**

6. Cauchy Estimates \( |f^{(n)}(z_0)| \leq \frac{n!M}{R^n} \).

Chapter 5 - Series

1. Geometric Series \( \sum_{k=0}^{\infty} w^k = \frac{1}{1 - w} \), \( |w| < 1 \).

2. Convergence Tests: **Ratio Test, Comparison Test, p-Series Test**.

3. \( c_k \not\to 0 \) Divergence Test.


5. Power series, region of convergence (obtained via convergence tests).

6. Operations on power series: addition, multiplication, differentiation, integration.

7. Computing Laurent series valid in certain regions (usually via Geometric series).

8. Isolated singularities: removable, pole, essential singularity.

9. Behavior of a function \( f(z) \) near \( \infty \) (consider \( f(\frac{1}{w}) \) near \( w = 0 \)).

see next page...
Chapter 6 - Residue Theory

1. Residue Theorem
2. Computing residues
3. Applications of residue Theorem: computing contour integrals; evaluating real integrals.