Convergence/Divergence of $\sum_{n=1}^{\infty} a_n$

$\lim_{n\to\infty} a_n = 0$ $\longrightarrow$ **NO** $\longrightarrow$ $\sum a_n$ **Diverges** by **Divergence Test**

$\downarrow$

YES

$\downarrow$

p-Series: $a_n = \dfrac{1}{n^p}$ $\longrightarrow$ **YES** $\longrightarrow$ $\sum \dfrac{1}{n^p} \begin{cases} \textbf{Converges}, & \text{if } p > 1 \\ \textbf{Diverges}, & \text{if } p \le 1 \end{cases}$

$\downarrow$

NO

$\downarrow$

Geometric Series: $a_n = a\, r^{n-1}$ $\longrightarrow$ **YES** $\longrightarrow$ $\sum_{n=1}^{\infty} a\, r^{n-1} \begin{cases} = \dfrac{a}{1-r} \ (\textbf{Converges}), & \text{if } |r| < 1 \\ \textbf{Diverges}, & \text{if } |r| \ge 1 \end{cases}$

$\downarrow$

NO

$\downarrow$

Alternating Series: $a_n = (-1)^{n-1} b_n$ $\longrightarrow$ **YES** $\longrightarrow$
- Use **Alternating Series Test**, or
- Use **Ratio/Root Test** to see if **Converges Absolutely**

$\downarrow$

NO

$\downarrow$

<u>Use Other Convergence Tests:</u>

(1) If a_n behaves like a p-series or Geometric series, usually use **Comparison Test** or use **Limit Comparison Test**. Find appropriate b_n to compare.

(2) If a_n involves factorials, usually use **Ratio Test**, $L = \lim_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right|$.

(3) If a_n involves n^{th} powers throughout, usually use **Root Test**, $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$.

(4) **Integral Test**.