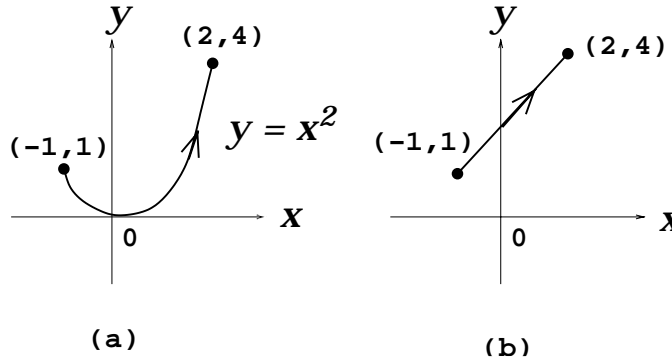


Supplementary Problems # 4

1. Page 113 : # 1 .

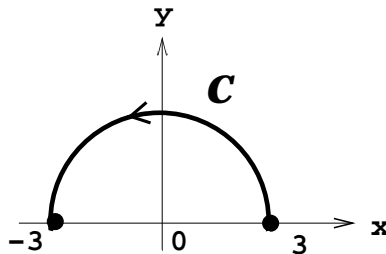
2. Page 122 : # 3(b)(d) .

3. Compute the integrals $\int_C 2xy dz$, $\int_C 2\bar{z} dz$, $\int_C 2z dz$, if C is the contour:



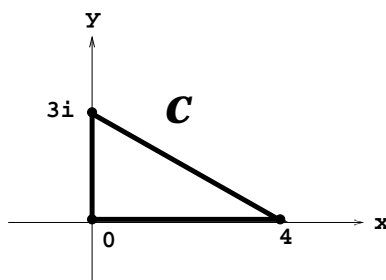
4. If C is the semicircle below and $f(z) = 2z$, evaluate these integrals

(a) $\int_C f(z) dz$ (b) $\int_C |f(z)| dz$ (c) $\int_C f(z) |dz|$

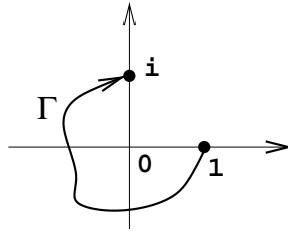


5. Page 123 : # 14(b) .

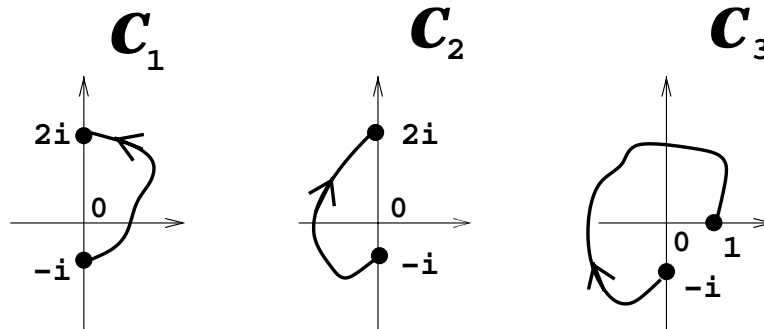
6. Show that $\left| \int_C \frac{z + e^{-z}}{z + 5} dz \right| \leq 60$, where C is the contour:



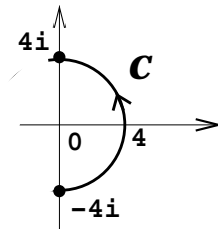
7. Compute $\int_{\Gamma} z e^{z^2} dz$ and $\int_{\Gamma} z e^z dz$ over the contour Γ :



8. Compute $\int_{C_1} \frac{1}{z} dz$, $\int_{C_2} \frac{1}{z} dz$, $\int_{C_3} \frac{1}{z} dz$ where



9. If $\sqrt{z} = e^{\frac{1}{2}\text{Log } z}$ is the principal branch of the square root, compute $\int_C \sqrt{z} dz$, where



10. Compute $\int_{|z|=4} \frac{\cos z}{z^2 - z} dz$ using Partial Fractions and using the Generalized Cauchy Theorem.

11. Evaluate $\int_{|z|=4} \frac{e^{\pi z}}{z(z-1)^2} dz$.

12. If C is the square with vertices at $\pm 2 \pm 2i$, traversed in a positive sense, compute these contour integrals:

$$(a) \int_C \frac{e^z + \sin z}{z(z-3)} dz \quad (b) \int_C \frac{z}{z^2 + 1} dz \quad (c) \int_C \frac{\cosh 2z}{(z+1)^4} dz.$$

13. **Page 167** : # 2, 3, 5, 6, 7 .

14. Does there exist a *nonconstant* function f which is analytic in $|z| \leq 2$, $f(0) = 2 + 3i$ and $|f(z)| \leq 3$ for all $|z| = 2$?

15. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D , prove that $\frac{\partial^2 u}{\partial x^2}$ is harmonic in D .

16. Page 161 : # 14 .

17. Page 168 : # 13 .

18. Show that the contour integral $\int_C f(z) dz$ is invariant under *nonsingular reparameterizations* of C i.e., If C has smooth parametrizations

$$C : z = z(t), \quad a \leq t \leq b$$

and

$$C : Z = z(\phi(\tau)), \quad \alpha \leq \tau \leq \beta,$$

where $\phi'(\tau) > 0$, $\phi(\alpha) = a$ and $\phi(\beta) = b$, then the integral $\int_C f(z) dz$ has the same value using either of these parameterizations of C .

19. POLYA VECTOR FIELDS - Physical Interpretation of Contour Integrals

Given any complex function $H(z) = u(x, y) + iv(x, y)$, the **Polya Vector Field** corresponding to $H(z)$ is given by

$$\vec{\mathbf{H}}(z) = (u(x, y), -v(x, y)).$$

Note that this is just the conjugate of H . Show that

$$\int_C H(z) dz = \int_C (\vec{\mathbf{H}} \cdot \vec{\mathbf{T}}) ds + i \int_C (\vec{\mathbf{H}} \cdot \vec{\mathbf{N}}) ds = (\text{WORK}) + i (\text{FLUX}),$$

where $\vec{\mathbf{T}}, \vec{\mathbf{N}}$ are the unit tangent and normal vectors to C at z . Hence the real part of $\int_C H(z) dz$ gives the total amount of work done in moving a particle along C subject to the force $\vec{\mathbf{H}}(z)$; while the imaginary part of $\int_C H(z) dz$ gives the total flux of the vector field $\vec{\mathbf{H}}(z)$ across C .