MATHEMATICS 182 TEST 3

(10 pts) 1) Find I_z, the moment of inertia with respect to the z–axis of the volume between $z = x^2 + y^2$ and $z = 1$ if the density $\delta = z$.

(10 pts) 2) Change the integral

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x dy dx$$

to polar coordinates and evaluate it.

(10 pts) 3) Find the area outside $r = 1$ and inside $r = 1 + \cos\theta$.

(10 pts) 4) Evaluate the line integral of $f(x, y, z) = x + y + z$ along the line connecting $(1, 1, 1)$ to $(2, 3, -2)$.

(10 pts) 5) Find the work done by the force $\vec{F} = x^2 i + z^2 k$ over the curve

$$x = \cos t \qquad y = \sin t \qquad z = t, \ 0 \leq t \leq \pi.$$

(10 pts) 6) Find the volume above $z = y^2$ and below $z = 4$ between $x = 0$ and $x = 1$.

(30 pts) 7) Set up but do not evaluate integrals for the following.

a) The mass of the tetrahedron with corners $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ if the density $\delta = 2y$,

b) The center of mass of the plate bounded by the parabola $y^2 = 4x$ and the line $x + y = 4$ if the density $\delta = 1$,

c) The volume between the spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 1$ above the cone $\varphi = \pi/4$.

(10 pts) 8) a) Solve the system

$$u = x + 2y \qquad \text{and} \qquad v = x - y \text{ for}$$

x and y. Find the Jacobian $J(u, v)$.

b) Sketch the region in the $x - y$ plane bounded by $y = 0$, $y = x$, and $x + 2y = 2$. What is the image in the $u - v$ plane?

c) Change $\int_0^{2/3} \int_y^{2-2y} (x + 2y)e^{y-x} dxdy$ into an integral over a domain in the $u - v$ plane. Do not evaluate the integral.