Math 182 Recitation2-7

Due at recitation, Thurs. Feb. 7, 2008

1. p. 1031, #12.

2. p. 1032, #40.

3. p. 1032, #43.

4. (a) Let $g(x, y, z)$ be a function whose gradient doesn't vanish at any point on the surface $g(x, y, z) = 0$. Let $Q = (a, b, c)$ be a point not on that surface. Let P be a point on the surface whose distance from Q is minimal, that is, $\leq$ the distance $P'Q$ for any other P' on the surface. Show that the line joining P and Q is perpendicular to the surface at P. (In other words, the sphere with center Q and passing through P is tangent to the surface at P.)

One way to proceed is to see what the Lagrange multiplier method says about minimizing the function $(x-a)^2 + (y-b)^2 + (z-c)^2$ with x, y, z constrained to satisfy $g(x, y, z) = 0$.

(b) Which point of the sphere $x^2 + y^2 + z^2 = 1$ has the greatest distance from $(1, 2, 3)$?

(c) In triangle ABC let the sides BC, AC, AB have lengths a, b, c, respectively. For a point P in the interior, let $x(P)$, $y(P)$, and $z(P)$ be the distances of P to BC, AC, and AB, respectively. Show that for the point where $x^2 + y^2 + z^2$ is minimal, it holds that

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{2\Delta}{a^2 + b^2 + c^2}$$

where Δ is the area of the triangle.

Hint. Begin by showing that for every P, $ax + by + cz = 2\Delta$. Then minimize $x^2 + y^2 + z^2$ subject to this restraint. You can do this with Lagrange multipliers; but there's an even easier way, using part (a).

5. Google "Lagrange multiplier" and also "Joseph Louis Lagrange."