PRACTICE EXAM 2

1. Let X have the density function $f(x) = \begin{cases} \frac{2x}{k^2} & \text{for } 0 \leq x \leq k \\ 0, & \text{otherwise} \end{cases}$.

For what value of k is the variance of X equal to 2?

A) 2 B) 6 C) 9 D) 18 E) 36

2. A life insurer classifies insurance applicants according to the following attributes:

M - the applicant is male

H - the applicant is a homeowner

Out of a large number of applicants the insurer has identified the following information:

40% of applicants are male, 40% of applicants are homeowners and

20% of applicants are female homeowners.

Find the percentage of applicants who are male and do not own a home.

A) 10% B) 20% C) 30% D) 40% E) 50%

3. Two components in an electrical circuit have continuous failure times X and Y. Both components will fail by time 1, but the circuit is designed so that the combined times until failure is also less than 1, so that the joint distribution of failure times satisfies the requirements $0 < x + y < 1$. How many of the following joint density functions are consistent with an expected combined time until failure less than $\frac{1}{2}$ for the two components?

I. $f(x, y) = 2$ II. $f(x, y) = 3(x + y)$ III. $f(x, y) = 6x$ IV. $f(x, y) = 6y$

A) 0 B) 1 C) 2 D) 3 E) 4

4. In a "wheel of fortune" game, the contestant spins a dial and it ends up pointing to a number uniformly distributed between 0 and 1 (continuous). After 10,000 independent spins of the wheel find the approximate probability that the average of the 10,000 spins is less than .499.

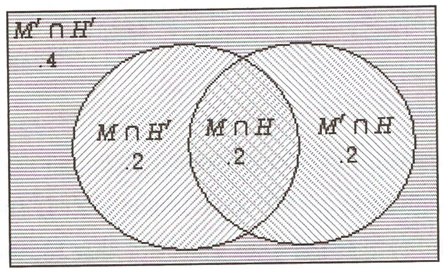
A) Less than .34 B) At least .34 but less than .35 C) At least .35 but less than .36

D) At least .36 but less than .37 E) At least .37

PRACTICE EXAM 2 - SOLUTIONS

1. $E[X] = \int_0^k x \cdot \frac{2x}{k^2}\,dx = \frac{2k}{3}$, $E[X^2] = \int_0^k x^2 \cdot \frac{2x}{k^2}\,dx = \frac{k^2}{2}$
$\Rightarrow\ Var[X] = \frac{k^2}{2} - (\frac{2k}{3})^2 = \frac{k^2}{18} = 2 \rightarrow k = 6$. Answer: B

2. $P[M] = .4$, $P[M'] = .6$, $P[H] = .4$, $P[H'] = .6$, $P[M' \cap H] = .2$,
We wish to find $P[M \cap H']$. From probability rules, we have
$.6 = P[H'] = P[M' \cap H'] + P[M \cap H']$, and
$.6 = P[M'] = P[M' \cap H] + P[M' \cap H'] = .2 + P[M' \cap H']$.
Thus, $P[M' \cap H'] = .4$ and then $P[M \cap H'] = .2$. The following diagram identifies the component probabilities.



The calculations above can also be summarized in the following table. The events across the top of the table categorize individuals as male (M) or female (M'), and the events down the left side of the table categorize individuals as homeowners (H) or non-homeowners (H').

	$P(M) = .4$, given		$P(M') = 1 - .4 = .6$
$P(H) = .4$ given	$P(M \cap H)$ $= P(H) - P(M' \cap H) = .4 - .2 = .2$ ⇓	⇐	$P(M' \cap H) = .2$, given
$P(H') = 1 - .4 = .6$	$P(M \cap H') = P(M) - P(M \cap H) = .4 - .2 = .2$		

Answer: B

3. $E[X+Y] = \int_0^1 \int_0^{1-x} (x+y) f(x,y)\,dy\,dx$
I. $\int_0^1 \int_0^{1-x} 2(x+y)\,dy\,dx = \frac{2}{3}$. Not correct.
II. $\int_0^1 \int_0^{1-x} 3(x+y)(x+y)\,dy\,dx = \frac{3}{4}$. Not correct.
III .$\int_0^1 \int_0^{1-x} 6x(x+y)\,dy\,dx = \frac{3}{4}$. Not correct.
IV .$\int_0^1 \int_0^{1-x} 6y(x+y)\,dy\,dx = \frac{3}{4}$. Not correct.
Note that III and IV will have the same outcome by the symmetry of x and y. Answer: A