19. Events X, Y and Z satisfy the following relationships:
$X \cap Y' = \phi$, $Y \cap Z' = \phi$, $P(X' \cap Y) = a$, $P(Y' \cap Z) = b$, $P(Z) = c$.
Find $P(X)$ in terms of a, b and c.
A) $a+b+c$ B) $a+b-c$ C) $c+b-a$ D) $c+a-b$ E) $c-b-a$

20. Let Z_1, Z_2, Z_3 be independent random variables each with mean 0 and variance 1, and let $X = 2Z_1 - Z_3$ and $Y = 2Z_2 + Z_3$. What is ρ_{XY} ?
A) -1 B) $-\frac{1}{3}$ C) $-\frac{1}{5}$ D) 0 E) $\frac{3}{5}$

21. Suppose that X has a binomial distribution based on 100 trials with a probability of success of .2 on any given trial. Find the approximate probability $P[15 \leq X \leq 25]$ using the integer correction.
A) .17 B) .34 C) .50 D) .67 E) .83

22. The model for the amount of damage to a particular property during a one-month period is as follows: there is a .99 probability of no damage, there is a .01 probability that damage will occur, and if damage does occur, it is uniformly distributed between 1000 and 2000. An insurance policy pays the amount of damage up to a policy limit of 1500. It is later found that the original model for damage when damage does occur was incorrect, and should have been uniformly distributed between 1000 and 5000. Find the amount by which the insurer's expected payment was understated when comparing the original model with the corrected model.
A) $\frac{11}{16}$ B) $\frac{13}{16}$ C) $\frac{15}{16}$ D) $\frac{17}{16}$ E) $\frac{19}{16}$

23. A coin is twice as likely to turn up tails as heads. If the coin is tossed independently, what is the probability that the third head occurs on the fifth toss?
A) $\frac{8}{81}$ B) $\frac{40}{243}$ C) $\frac{16}{81}$ D) $\frac{80}{243}$ E) $\frac{3}{5}$

19. $X = (X \cap Y) \cup (X \cap Y') \rightarrow X = X \cap Y \rightarrow$
$P(Y) = P(Y \cap X) + P(Y \cap X') = P(X) + a.$
$Y = (Y \cap Z) \cup (Y \cap Z') \rightarrow Y = Y \cap Z \rightarrow c = P(Z) = P(Z \cap Y) + P(Z \cap Y') = P(Y) + b.$
Then, $P(X) + a + b = c \rightarrow P(X) = c - b - a$. {}It is also true that $X \subset Y \subset Z$, so that
$c = P(Z) = P(X) + P(Y - X) + P(Z - Y)$
$= P(X) + P(Y \cap X') + P(Z \cap Y') = P(X) + a + b$. Answer: E

20. $\rho_{XY} = \frac{Cov[X,Y]}{\sigma_X \sigma_Y} = \frac{Cov[2Z_1 - Z_3, 2Z_2 + Z_3]}{\sqrt{Var[2Z_1 - Z_3] \cdot Var[2Z_2 + Z_3]}}$.
$Cov[2Z_1 - Z_3, 2Z_2 + Z_3]$
$= 4Cov[Z_1, Z_2] + 2Cov[Z_1, Z_3] - 2Cov[Z_3, Z_2] - Cov[Z_3, Z_3]$
$= 4(0) + 2(0) - 2(0) - Var[Z_3] = -1$ (since $Cov[Z_3, Z_3] = Var[Z_3]$ and independent random variables have covariance of 0).
$Var[2Z_1 - Z_3] = 4Var[Z_1] + Var[Z_3] - 2(2Cov[Z_1, Z_3]) = 5$,
$Var[2Z_2 + Z_3] = 4Var[Z_2] + Var[Z_3] + 2(2Cov[Z_2, Z_3]) = 5$.
Thus, the correlation is $\rho_{XY} = \frac{-1}{\sqrt{(5) \cdot (5)}} = -\frac{1}{5}$. Answer: C

21. The mean and variance of X are $E[X] = 100(.2) = 20$, $Var[X] = 100(.2)(.8) = 16$. Using the normal approximation with integer correction, we assume that X is approximately normal and find
$P[14.5 \leq X \leq 25.5] = P[\frac{14.5-20}{\sqrt{16}} \leq \frac{X-20}{\sqrt{16}} \leq \frac{25.5-20}{\sqrt{16}}] = P[-1.375 \leq Z \leq 1.375]$,
where Z has a standard normal distribution.
$P[-1.375 \leq Z \leq 1.375] = \Phi(1.375) - \Phi(-1.375) = \Phi(1.375) - [1 - \Phi(1.375)]$
$= 2\Phi(1.375) - 1$.
From the standard normal table we have $\Phi(1.3) = .9032$ and $\Phi(1.4) = .9192$. Using linear interpolation (since 1.375 is $\frac{3}{4}$ of the way from 1.3 to 1.4) we have
$\Phi(1.375) = (.25)\Phi(1.3) + (.75)\Phi(1.4) = .9152$, and then the probability in question is
$2(.9152) - 1 = .8304$. Answer: E