5. Let A and B be events such that $P[A] = .7$ and $P[B] = .9$.
Calculate the largest possible value of $P[A \cup B] - P[A \cap B]$.
A) .20 B) .34 C) .40 D) .60 E) 1.60

6. Bob has a fair die and tosses it until a "1" appears. Doug also has a fair die, and he tosses it until a "1" appears. Joe also has a fair die and he tosses it until a "1" appears. They each stop tossing as soon as a "1" turns up on their die. We define the random variable X to be the total number of tosses that Bob, Doug and Joe made (including the first "1" that each of them tossed). Find $Var[X]$.
A) 50 B) 60 D) 70 D) 80 E) 90

7. Let X and Y be independent continuous random variables with common density function

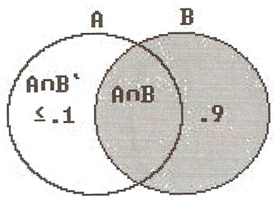
$$f(t) = \begin{cases} 1 \text{ for } 0<t<1 \\ 0, \text{otherwise} \end{cases} .$$

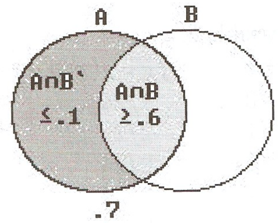
What is $P[X^2 \geq Y^3]$?
A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{2}{3}$ E) 1

8. An auto insurer's portfolio of policies is broken into two classes - low risk, which make up 75% of the policies, and high risk, which make up 25% of the policies. The number of claims per year that occur from a policy in the low risk group has a Poisson distribution with a mean of .2, and the number of claims per year that occur from a policy in the high risk group has a Poisson distribution with a mean of 1.5 . A policy is chosen at random from the insurer's portfolio. Find the probability that there will be exactly one claim during the year on that policy.
A) .21 B) .25 C) .29 D) .33 E) .37

9. Every member of an insured group has an annual claim amount distribution that is exponentially distributed. The expected claim amount of a randomly chosen member of the group is $\frac{1}{c}$, where c is uniformly distributed between 1 and 2. Find the probability that a randomly chosen member of the group has annual claim less than 1.
A) Less than .4 B) At least .4 but less than .5 C) At least .5 but less than .6
D) At least .6 but less than .7 E) At least .7

5. $P[A\cup B]-P[A\cap B]=P[A]+P[B]-2P[A\cap B]=1.6-2P[A\cap B]$.
This will be maximized if $P[A\cap B]$ is minimized.
But $.7=P[A]=P[A\cap B']+P[A\cap B]$, and the maximum possible value of $P[A\cap B']$ is .1 (since $P[B]$ is .9, it follows that $P[A\cap B']\leq P[B']=.1$), so that the minimum possible value for $P[A\cap B]$ is .6, and then the maximum of $P[A\cup B]-P[A\cap B]$ is $1.6-2(.6)=.4$.





Answer: C

6. If we consider Doug first and define "success" to be tossing a "1" and "failure" to be a toss that is not a "1" then the number of tosses Doug makes, say X, before his first 1 is the number of failures before the first success. X has a geometric distribution with $p=\frac{1}{6}$ (p is the probability of success on a single trial). Doug's total number of tosses is $X+1$, and
$Var(X+1)=Var(X)=\frac{1-p}{p^2}=\frac{1-\frac{1}{6}}{(\frac{1}{6})^2}=30$.
Bob and Joe have the same distribution for the number of tosses until a first "1", and since they toss independently of Doug and each other, the variance of the total number of tosses is just the sum of the three variances, which is $30+30+30=90$. Answer: E

7. Since both X and Y are between 0 and 1, the event $X^2>Y^3$ is equivalent to $X>Y^{3/2}$. Since X and Y are independent, their joint density is
$f(x,y)=f_X(x)\cdot f_Y(y)=1$. Then,
$P[X^2>Y^3]=\int_0^1\int_{y^{3/2}}^1 1\,dx\,dy=\int_0^1(1-y^{3/2})\,dy=\frac{3}{5}$. Answer: C

8. $P[N=1]=P[N=1|\text{low risk}]\cdot P[\text{low risk}]+P[N=1|\text{high risk}]\cdot P[\text{high risk}]$
$P[N=1|\text{low risk}]=e^{-.2}(.2)$, $P[N=1|\text{high risk}]=e^{-1.5}(1.5)$,
$P[\text{low risk}]=.75$, $P[\text{high risk}]=.25$
$\rightarrow P[N=1]=(.75)e^{-.2}(.2)+(.25)e^{-1.5}(1.5)=.206$. Answer: A