PROBLEM SET 1

Basic Probability Concepts

1. A survey of 1000 people determines that 80% like walking and 60% like biking, and all like at least one of the two activities. What is the probability that a randomly chosen person in this survey likes biking but not walking?

A) 0 B) .1 C) .2 D) .3 E) .4

2. A survey of 1000 Canadian sports fans who indicated they were either hockey fans or lacrosse fans or both, had the following result.

- 800 indicated that they were hockey fans
- 600 indicated that they were lacrosse fans

Based on the sample, find the probability that a Canadian sports fan is not a hockey fan given that she/he is a lacrosse fan.

A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) 1

3. (SOA) Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.

A) 0.26 B) 0.38 C) 0.40 D) 0.48 E) 0.62

4. (SOA) An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44 Calculate the number of blue balls in the second urn.

A) 4 B) 20 C) 24 D) 44 E) 64

5. (SOA) An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{5}{12}$, respectively. Determine the probability that a randomly chosen employee will choose no supplementary coverage.

A) 0 B) $\frac{47}{144}$ C) $\frac{1}{2}$ D) $\frac{97}{144}$ E) $\frac{7}{9}$

6. (SOA) An auto insurance company has 10,000 policyholders.
Each policyholder is classified as
(i) young or old;
(ii) male or female; and
(iii) married or single.
Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

A) 280 B) 423 C) 486 D) 880 E) 896

7. (SOA) An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is $\frac{1}{3}$. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

A) 0.280 B) 0.311 C) 0.467 D) 0.484 E) 0.700

8. (SOA) The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35% . Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

A) 0.05 B) 0.12 C) 0.18 D) 0.25 E) 0.35

9. (SOA) You are given $P[A \cup B] = 0.7$ and $P[A \cup B'] = 0.9$. Determine $P[A]$.

A) 0.2 B) 0.3 C) 0.4 D) 0.6 E) 0.8

10. (SOA) A survey of a group's viewing habits over the last year revealed the following information:

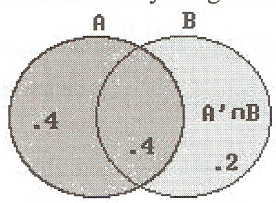
(i) 28% watched gymnastics
(ii) 29% watched baseball
(iii) 19% watched soccer
(iv) 14% watched gymnastics and baseball
(v) 12% watched baseball and soccer
(vi) 10% watched gymnastics and soccer
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

A) 24 B) 36 C) 41 D) 52 E) 60

PROBLEM SET 1 SOLUTIONS

1. Let $A =$ "like walking" and $B =$ "like biking" . We use the interpretation that "percentage" and "proportion" are taken to mean "probability".
We are given $P(A) = .8$, $P(B) = .6$ and $P(A \cup B) = 1$.
From the diagram below we can see that since $A \cup B = A \cup (B \cap A')$ we have
$P(A \cup B) = P(A) + P(A' \cap B) \rightarrow P(A' \cap B) = .2$ is the proportion of people who like biking but (and) not walking . In a similar way we get $P(A \cap B') = .4$.



An algebraic approach is the following. Using the rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, we get $1 = .8 + .6 - P(A \cap B) \rightarrow P(A \cap B) = .4$. Then, using the rule $P(B) = P(B \cap A) + P(B \cap A')$, we get $P(B \cap A') = .6 - .4 = .2$. Answer: C

2. From the given information, 400 of those surveyed are both hockey and lacrosse fans, 200 are lacrosse fans and not hockey fans, and 400 are hockey fans an not lacrosse fans. This is true because there are 1000 fans in the survey, but a combined total of $800 + 600 = 1400$ sports preferences, so that 400 must be fans of both. Of the 600 lacrosse fans, 400 are also hockey fans, so 200 are not hockey fans The probability that a Canadian sports fans is not a hockey fan given that she/he is a lacrosse fan is $\frac{200}{600} = \frac{1}{3}$. Answer: C

3. C - chiropractor visit ; T - therapist visit.
We are given $P(C \cap T) = .22$, $P(C' \cap T') = .12$, $P(C) = P(T) + .14$.
$.88 = 1 - P(C' \cap T') = P(C \cup T) = P(C) + P(T) - P(C \cap T)$
$= P(T) + .14 + P(T) - .22 \rightarrow P(T) = .48$. Answer: D

4. Suppose there are B blue balls in urn II.
P[both balls are same color] $= P$[both blue $\cup$ both red] $= P$[both blue] $+ P$[both red]
(the last equality is true since the events "both blue" and "both red" are disjoint).
P[both blue] $= P$[blue from urn I $\cap$ blue from urn II]
$= P$[blue from urn I] $\cdot P$[blue from urn II] (choices from the two urns are independent)
$= (\frac{6}{10})(\frac{B}{16+B})$,
P[both red] $= P$[red from urn I $\cap$ red from urn II]
$= P$[red from urn I] $\cdot P$[red from urn II] $= (\frac{4}{10})(\frac{16}{16+B})$,
We are given $(\frac{6}{10})(\frac{B}{16+B}) + (\frac{4}{10})(\frac{16}{16+B}) = .44 \rightarrow \frac{6B+64}{10(16+B)} = .44 \rightarrow B = 4$.
Answer: A

5. Since someone who chooses coverage must choose exactly two supplementary coverages, in order for someone to choose coverage A, they must choose either A-and-B or A-and-C. Thus, the proportion of $\frac{1}{4}$ of individuals that choose A is
$P[A \cap B] + P[A \cap C] = \frac{1}{4}$ (where this refers to the probability that someone chosen at random in the company chooses coverage A). In a similar way we get
$P[B \cap A] + P[B \cap C] = \frac{1}{3}$ and $P[C \cap A] + P[C \cap B] = \frac{5}{12}$.
Then, $(P[A \cap B] + P[A \cap C]) + (P[B \cap A] + P[B \cap C]) + (P[C \cap A] + P[C \cap B])$
$= 2(P[A \cap B] + P[A \cap C] + P[B \cap C]) = \frac{1}{4} + \frac{1}{3} + \frac{5}{12} = 1$.
It follows that $P[A \cap B] + P[A \cap C] + P[B \cap C] = \frac{1}{2}$.
This is the probability that a randomly chosen individual chooses some form of coverage, since if someone who chooses coverage chooses exactly two of A,B, and C. Therefore, the probability that a randomly chosen individual does not choose any coverage is the probability of the complementary event, which is also $\frac{1}{2}$. Answer: C

6. We identify the following subsets of the set of 10,000 policyholders:
$Y =$ young, with size 3000 (so that $Y' =$ old has size 7000),
$M =$ male, with size 4600 (so that $M' =$ female has size 5400), and
$C =$ married, with size 7000 (so that $C' =$ single has size 3000).
We are also given that $Y \cap M$ has size 1320, $M \cap C$ has size 3010,
$Y \cap C$ has size 1400, and $Y \cap M \cap C$ has size 600.
We wish to find the size of the subset $Y \cap M' \cap C'$.
We use the following rules of set theory:

6. continued

(i) if two finite sets are disjoint (have no elements in common, also referred to as empty intersection), then the total number of elements in the union of the two sets is the sum of the numbers of elements in each of the sets;

(ii) for any sets A and B, $A = (A \cap B) \cup (A \cap B')$, and $A \cap B$ and $A \cap \overline{B}$ are disjoint.

Applying rule (ii), we have $Y = (Y \cap M) \cup (Y \cap M')$. Applying rule (i), it follows that the size of $Y \cap M'$ must be $3000 - 1320 = 1680$.

We now apply rule (ii) to $Y \cap C$ to get $Y \cap C = (Y \cap C \cap M) \cup (Y \cap C \cap M')$.

Applying rule (i), it follows that $Y \cap C \cap M'$ has size $1400 - 600 = 800$.

Now applying rule (ii) to $Y \cap M'$ we get $Y \cap M' = (Y \cap M' \cap C) \cup (Y \cap M' \cap C')$.

Applying rule (i), it follows that $Y \cap M' \cap C'$ has size $1680 - 800 = 880$.

Within the "Young" category, which we are told is 3000, we can summarize the calculations in the following table. This is a more straightforward solution.

	Married 1400 (given)	Single 1600 = 3000 − 1400
Male 1320 (given)	$\boxed{600 \text{ (given)}}$	$\boxed{720 = 1320 - 600}$
Female 1680 = 3000 − 1320	$\boxed{800 = 1400 - 600}$	$\boxed{880 = 1600 - 720}$

Answer: D

7. We are given

$P[A \cap B' \cap C'] = P[A' \cap B \cap C'] = P[A' \cap B' \cap C] = .1$

(having exactly one risk factor means not having either of the other two).

We are also given

$P[A \cap B \cap C'] = P[A \cap B' \cap C] = P[A' \cap B \cap C] = .12$.

And we are given

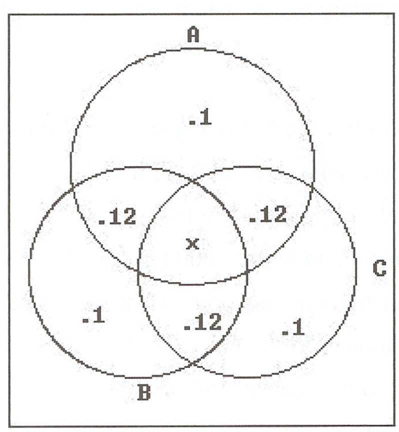
$P[A \cap B \cap C | A \cap B] = \frac{1}{3}$.

We are asked to find $P[A' \cap B' \cap C' | A']$.

From $P[A \cap B \cap C | A \cap B] = \frac{1}{3}$ we get $\frac{P[A \cap B \cap C]}{P[A \cap B]} = \frac{1}{3}$, and then

$P[A \cap B \cap C] = \frac{1}{3} \cdot P[A \cap B]$.

The following Venn diagram illustrates the situation:



7. continued

We see that $P[A \cap B \cap C] = x$ and $P[A \cap B] = x + .12$, so that

$x = \frac{1}{3} \cdot (x + .12) \rightarrow x = P[A \cap B \cap C] = .06$.

Alternatively, we can use the rule $P[D] = P[D \cap E] + P[D \cap E']$ to get

$P[A \cap B] = P[A \cap B \cap C] + P[A \cap B \cap C'] = P[A \cap B \cap C] + .12$.

Then, $P[A \cap B] = P[A \cap B \cap C] + .12 = \frac{1}{3} \cdot P[A \cap B] + .12 \rightarrow P[A \cap B] = .18$

and $P[A \cap B \cap C] = \frac{1}{3} \cdot (.18) = .06$.

We can also see from the diagram that $P[A \cap B'] = .1 + .12 = .22$.

Alternatively, we can use the rule above again to get

$P[A \cap B'] = P[A \cap B' \cap C] + P[A \cap B' \cap C'] = .12 + .1 = .22$.

Then, $P[A] = P[A \cap B] + P[A \cap B'] = .18 + .22 = .4$, and $P[A'] = 1 - P[A] = .6$.

We are asked to find $P[A' \cap B' \cap C' | A'] = \frac{P[A' \cap B' \cap C']}{P[A']} = \frac{P[A' \cap B' \cap C']}{.6}$, so we must find

$P[A' \cap B' \cap C']$. From the Venn diagram, we see that

$P[A' \cap B' \cap C'] = 1 - (.1 + .1 + .1 + .12 + .12 + .12 + .06) = .28$.

Finally, $P[A' \cap B' \cap C' | A'] = \frac{P[A' \cap B' \cap C']}{P[A']} = \frac{P[A' \cap B' \cap C']}{.6} = \frac{.28}{.6} = .467$. Answer: C

8. We identify events as follows:

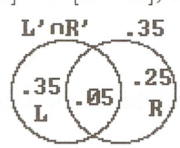
L : lab work needed

R : referral to a specialist needed

We are given $P[L' \cap R'] = .35$, $P[R] = .3$, $P[L] = .4$. It follows that

$P[L \cup R] = 1 - P[L' \cap R'] = .65$, and then since

$P[L \cup R] = P[L] + P[R] - P[L \cap R]$, we get $P[L \cap R] = .3 + .4 - .65 = .05$.

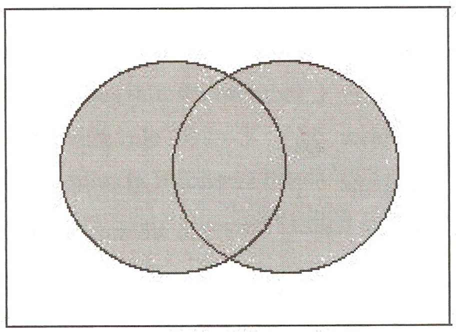


These calculations can be summarized in the following table

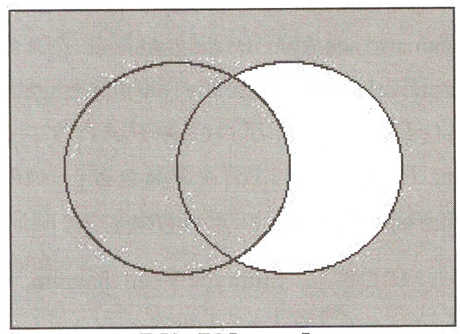
	L , .4 given	L' , .6 $.6 = 1 - .4$
R , .3 given	$L \cap R$ $.05 = .4 - .35$	$L' \cap R$ $.25 = .3 - .05$
R' , .7 $.7 = 1 - .3$	$L \cap R'$ $.35 = .7 - .35$	$L' \cap R'$, .35 given

Answer: A

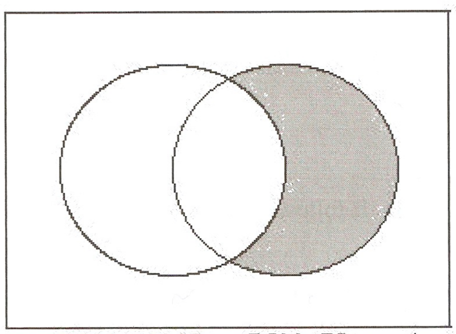
9. $P[A \cup B] = P[A] + P[B] - P[A \cap B]$, $P[A \cup B'] = P[A] + P[B'] - P[A \cap B']$.
We use the relationship $P[A] = P[A \cap B] + P[A \cap B']$. Then
$P[A \cup B] + P[A \cup B'] = P[A] + P[B] - P[A \cap B] + P[A] + P[B'] - P[A \cap B']$
$= 2P[A] + 1 - P[A] = P[A] + 1$ (since $P[B] + P[B'] = 1$) .
Therefore, $.7 + .9 = P[A] + 1$ so that $P[A] = .6$.
An alternative solution is based on the following Venn diagrams.



P[AuB] = .7



P[AuB'] = .9



P[(AuB')'] = P[A'nB] = .1

In the third diagram, the shaded area is the complement of that in the second diagram (using De Morgan's Law, we have $(A \cup B')' = A' \cap B'' = A' \cap B$). Then it can be seen from diagrams 1 and 3 that $A = (A \cup B) - (A' \cap B)$, so that
$P[A] = P[A \cup B] - P[A' \cap B] = .7 - .1 = .6$. Answer: D

