27. The value, v, of an appliance is based on the number of years since purchase, t, as follows: $v(t) = e^{(7-.2t)}$. If the appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance. After seven years the warranty pays nothing. The time until failure of the appliance has an exponential distribution with a mean of 10. Calculate the expected payment from the warranty.

A) 98.70 B) 109.66 C) 270.43 D) 320.78 E) 352.16

28. A test for a disease correctly diagnoses a diseased person as having the disease with probability .85. The test incorrectly diagnoses someone without the disease as having the disease with a probability of .10. If 1% of the people in a population have the disease, what is the chance that a person from this population who tests positive for the disease actually has the disease?

A) .0085 B) .0791 C) .1075 D) .1500 E) .9000

29. Let X and Y be discrete random variables with joint probability function $f(x,y)$ given by the following table:

		x			
		2	3	4	5
	0	.05	.05	.15	.05
y	1	.40	0	0	0
	2	.05	.15	.10	0

For this joint distribution, $E[X] = 2.85$ and $E[Y] = 1$. Calculate $Cov[X,Y]$.

A) $-.20$ B) $-.15$ C) .95 D) 2.70 E) 2.85

30. One of the questions asked by an insurer on an application to purchase a life insurance policy is whether or not the applicant is a smoker. The insurer knows that the proportion of smokers in the general population is .30, and assumes that this represents the proportion of applicants who are smokers. The insurer has also obtained information regarding the honesty of applicants:

- 40% of applicants that are smokers say that they are non-smokers on their applications,
- none of the applicants who are non-smokers lie on their applications.

What proportion of applicants who say they are non-smokers are actually non-smokers?

A) 0 B) $\frac{6}{41}$ C) $\frac{12}{41}$ D) $\frac{35}{41}$ E) 1

24. $f(\alpha) = \int_\alpha^C (x-\alpha)\cdot\frac{1}{C}\,dx = \frac{(C-\alpha)^2}{2C} \rightarrow f'(\alpha) = \frac{\alpha}{C} - 1$. Answer: D

25. Because of independence, $P[(K=3)\cap(L=6)] = P[K=3]\cdot P[L=6]$
$= \left[\binom{5}{3}(.3)^3(.7)^2\right]\left[\binom{10}{6}(.1)^6(.9)^4\right]$
(K and L both have binomial distributions). Answer: B

26. $f(x) = \begin{cases} x & x \le 750 \\ 750 & x > 750 \end{cases} \rightarrow f'(x) = \begin{cases} 1 & x \le 750 \\ 0 & x > 750 \end{cases}$.
This is the graph in C. Answer: C

27. The expected value of the warranty is $E[w(m)] = \int_0^\infty w(m)\cdot f(m)\,dm$, where $f(m)$ is the density function of the appliance failing at time m. We are given that the failure time has an exponential distribution with a mean of 10. The mean of an exponential distribution with parameter λ is $\frac{1}{\lambda} = 10$, so that $\lambda = .1$. and the density function is $f(m) = .1e^{-.1m}$. The expected value of the warranty is $E[w(m)] = \int_0^7 v(m)\cdot .1e^{-.1m}\,dm = \int_0^7 e^{(7-.2m)}\cdot .1e^{-.1m}\,dm$
$= .1e^7\int_0^7 e^{-.3m}dm = .1e^7[\frac{1-e^{-2.1}}{.3}] = 320.78$. Answer: D

28. We define the following events: D - a person has the disease ,
TP - a person tests positive for the disease. We are given $P[TP|D] = .85$ and $P[TP|D'] = .10$ and $P[D] = .01$. We wish to find $P[D|TP]$.

With a model population of 10,000, there would be $10,000 \times P(D) = 100 = \#D$ people with the disease and 9,900 without the disease. The number that have the disease and test positive is $\#D\cap TP = \#D \times P[TP|D] = 100\times .85 = 85$ and the number that do not have the disease and test positive is $\#D'\cap TP = \#D' \times P[TP|D'] = 9,900\times .1 = 990$. The total number who test positive is $\#D = \#D\cap TP + \#D'\cap TP = 85 + 990 = 1075$. The probability that someone who tests positive actually has the disease is the proportion $\frac{\#D\cap TP}{TP} = \frac{85}{1075} = .0791$.

The conditional probability approach to solving the problem is as follows.
Using the formulation for conditional probability we have $P[D|TP] = \frac{P[D\cap TP]}{P[TP]}$.
But $P[D\cap TP] = P[TP|D]\cdot P[D] = (.85)(.01) = .0085$, and
$P[D'\cap TP] = P[TP|D']\cdot P[D'] = (.10)(.99) = .099$. Then,
$P[TP] = P[D\cap TP] + P[D'\cap TP] = .1075 \rightarrow P[D|TP] = \frac{.0085}{.1075} = .0791$.

28. continued

The following table summarizes the calculations.

$P[D] = .01$, given $\Rightarrow$ $P[D'] = 1 - P[D] = .99$

$\Downarrow$ $\Downarrow$

$P[D \cap TP]$ $P[D' \cap TP]$

$= P[TP|D] \cdot P[D] = .0085$ $= P[TP|D'] \cdot P[D'] = .099$

$\Downarrow$

$P[TP] = P[D \cap TP] + P[D' \cap TP] = .1075$

$\Downarrow$

$P[D|TP] = \frac{P[D \cap TP]}{P[TP]} = \frac{.0085}{.1075} = .0791$. Answer: B

29. $Cov[X,Y] = E[XY] - E[X] \cdot E[Y] = E[XY] - 2.85$.

$E[XY] = \sum_{x=2}^{5} \sum_{y=0}^{2} xy \cdot f(x,y) = 2 \cdot 0 \cdot (.05) + 2 \cdot 1 \cdot (.40) + \cdots + 5 \cdot 1 \cdot (0) + 5 \cdot 2 \cdot (0)$

$= 2.7 - 2.85 = -.15$. Answer: B

30. We identify the following events:

S - the applicant is a smoker, NS - the applicant is a non-smoker $= S'$

DS - the applicant declares to be a smoker on the application

DN - the applicant declares to be non-smoker on the application $= DS'$.

The information we are given is $P[S] = .3$, $P[NS] = .7$, $P[DN|S] = .4$, $P[DS|NS] = 0$.

We wish to find $P[NS|DN] = \frac{P[NS \cap DN]}{P[DN]}$.

With a model population of 100 there are $30 = \#S$ smokers and $70 = \#NS$ non-smokers. The number of smokers who declare that they are non-smokers is $\#DN \cap S = \#S \times P[DN|S] = 30 \times .4 = 12$ and since non-smokers don't lie, the number of non-smokers who declare that they are non-smokers is equal to the number of non-smokers, so $\#DN \cap NS = NS = 70$. The total number of people who declare that they are non-smokers is $\#DN \cap S + \#DN \cap NS = 12 + 70 = 82 = \#DN$.

Then, the proportion of applicants who say they are non-smokers that are actually non-smokers is $\frac{\#DN \cap NS}{DN} = \frac{70}{82} = \frac{35}{41}$.

The conditional probability approach to the solution is on the next page.

