13. In a survey of males over the age of 30, it is found 50% are married, 40% smoke, 30% own a home and 60% own a car. It is also found that 30% are non-smoking bachelors, 40% are married car owners, 36% are non-smoking car owners, 25% own both a home and a car and 20% are married and own a home and a car. Which of the following statements is true regarding independence among the attributes of being married, being a smoker, being a car owner and being a home owner?

A) Being single and owning a car are independent

B) Being married and smoking are not independent

C) Being a smoker and owning a car are independent

D) Being a home owner and being a car owner are independent

E) Being married, being a home owner and being a car owner are mutually independent

14. A loss random variable is uniformly distributed on the integers from 0 to 11.
An insurance pays the loss in excess of a deductible of 5.5 . Find the expected amount not covered by the insurance.

A) 2 B) 3 C) 4 D) 5 E) 6

15. An insurer offers an "all or nothing" policy of the following type. If the loss being insured is for an amount of D or more, then the insurance policy pays the full amount, but if the loss is less than D then the policy pays nothing. Assuming that the distribution of the loss has an exponential distribution with a mean of 2, and that $D = 2$, find the expected payout on the policy.

A) $\frac{1}{e}$ B) $\frac{2}{e}$ C) $\frac{4}{e}$ D) e E) $2e$

16. Suppose that X is a random variable with moment generating function $M(t) = \sum_{j=0}^{\infty} \frac{e^{(tj-1)}}{j!}$. Find $P[X = 2]$.

A) 0 B) $\frac{1}{2e}$ C) $\frac{e}{2}$ D) $\frac{1}{2}$ E) $\sum_{j=0}^{\infty} \frac{e^{2j-1}}{j!}$

13. M - married, S - smoker, C - car owner, H - home owner

We are given, $P[M] = .5$, $P[S] = .4$, $P[C] = .6$, $P[H] = .3$, $P[M' \cap S'] = .3$, $P[M \cap C] = .4$, $P[S' \cap C] = .36$, $P[H \cap C] = .25$, $P[M \cap C \cap H] = .2$.

Then, $P[M' \cap C] = P[C] - P[M \cap C] = .6 - .4 = .2$,

but $P[M'] \cdot P[C] = (.5)(.6) = .3 \rightarrow M', C$ not independent $\rightarrow$ A is false.

$P[M' \cap S'] = .3$, $P[M'] \cdot P[S'] = (.5)(.6) = .3 \rightarrow M', S'$ are independent

$\rightarrow M, S$ are independent $\rightarrow$ B is false.

$P[S' \cap C] = .36 = (.6)(.6) = P[S'] \cdot P[C] \rightarrow S', C$ are independent

$\rightarrow S, C$ are independent $\rightarrow$ C is true.

We can also check $P[H \cap C] = .25 \neq (.3)(.6) = P[H] \cdot P[C]$

$\rightarrow H, C$ not independent $\rightarrow$ D is false,

$P[M \cap C \cap H] = .2 \neq (.5)(.6)(.3) = P[M] \cdot P[C] \cdot P[H]$

$\rightarrow M, C, H$ are not mutually independent. Answer: C

14. The amount not covered is

Loss	0	1	2	3	4	5	6	7	$\cdots$	11
Amt. Not Covered	0	1	2	3	4	5	5.5	5.5	$\cdots$	5.5
Prob.	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\cdots$	$\frac{1}{12}$

The expected amount not covered by the insurance is

$(\frac{1}{12})[0 + 1 + 2 + 3 + 4 + 5 + 5.5(6)] = 4$. Answer: C

15. The expected payment on the policy will be

$\int_2^\infty x \cdot \frac{1}{2} e^{-x/2}\, dx = -xe^{-x/2} - 2e^{-x/2}\Big|_{x=2}^{x=\infty} = 4e^{-1}$. Answer: C

16. The moment generating function for a non-negative discrete integer-valued random variable X with probability function f is defined to be $M(t) = E[e^{tX}] = \sum_{j=0}^{\infty} e^{tj} \cdot f(j)$.

Since we are given that $M(t) = \sum_{j=0}^{\infty} \frac{e^{(tj-1)}}{j!}$, and it is known that the distribution of a random variable is uniquely determined by its moment generating function (i.e., there is precisely one probability distribution with that specified mgf), it follows that

$f(j) = \frac{e^{-1}}{j!} = \frac{1}{e \cdot j!}$. Since $f(j) = P[X = j]$, it follows that $P[X = 2] = \frac{1}{2e}$.

Answer: B