

Practice Midterm 1

MA 266

Lecturer: Javier

NAME: _____

No calculators or notes allowed. Show your work.

1. (10 points) Determine the order of the given differential equation and state whether it is linear or non-linear.

(a) $y''' + y'' + y = 1$

(b) $y'' + \sin(t + y) = \cos t$

(c) $y' + ty^2 = 0$

(d) $(y')^{yy''} + \frac{1}{t^2} = 0$

(e) $t^3 y''' + \frac{y'}{\cos t} = e^{\sqrt{t}}$

2. (10 points) Find the implicit solution of

$$\frac{dy}{dx} = \frac{x^2 + \cos x}{e^y + y}$$

3. (10 points) Given the equation

$$y' = y(9 - y^2)$$

(a) Sketch the direction field

(b) Find the equilibrium solutions and state their stability

4. (25 points) Find the solution to the initial value problem

$$ty' + 3y = \frac{1}{t^2 + t^4}$$

with initial condition $y(1) = \pi/2$. Find also the biggest interval where this solution is defined.

5. (15 points) Given the initial value problem

$$y' = 3 + t - y$$

with $y(0) = 1$ use Euler's method to approximate $y(3)$ with steps of length one.

6. (25 points) Find the general solution of

$$(y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$$

7. (5 points) Initially a tank holds 50 gal of pure water. A salt solution containing 1/2 lb of salt per gal runs into the tank at the rate of 10 gal per minute. The well mixed solution runs out of the tank at the same rate. Let $S(t)$ be the amount of salt in the tank at time t . Find a differential equation satisfied by $S(t)$ (Do not solve the equation).