

Practice Midterm 3

MA 266

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No calculators or notes allowed. Show your work. You can use the Table of Elementary Laplace Transforms on Page 319 of your textbook.

1. (20 points) Solve the initial value problem

$$y''' - y'' + y' - y = 1 + 4e^{-t}$$

with initial conditions  $y(0) = 1$ ,  $y'(0) = -1$ ,  $y''(0) = 0$ .

2. (10 points) Sketch the graph of the function:

(a)  $1 - u_1(t)$

(b)  $u_1(t) - u_2(t)$

(c)  $u_1(t) + u_2(t)$

(d)  $u_1(t) \cdot u_2(t)$

(e)  $u_1(t) \cdot u_1(t)$

3. (15 points) Consider the IVP  $y'' + y = f(t)$  with initial conditions  $y(0) = 0$ ,  $y'(0) = 0$ . If the solution is found to be  $u_1(t) \sin(t - 1) + u_2(t)(1 - \cos(t - 2))$  find the forcing function  $f(t)$ . (Brownie points: What letter of the alphabet is the forcing function describing?)

4. (20 points) Find  $f(t)$  if

(a)  $f(t) - \int_0^t (t - x)f(x)dx = 1$

(b)  $f(t) * 1 = f(t) + 1$

5. (15 points) Suppose we have three tanks of water. Tank 1 initially contains 40 gals of water with 10 oz of salt in it. Tank 2 initially contains 50 gals of water with 5 oz of salt in it. Tank 3 initially contains 60 gals of pure water. Water containing 5 oz/gal of salt flows into Tank 1 at a rate of 10 gal/min and the well-stirred mixture flows from Tank 1 to Tank 2 and Tank 3 at a rate of 6 gal/min and 4 gal/min respectively. The solution on Tanks 2 and 3 flows out to the ground at a rate of 6 gal/min and 4 gal/min respectively. Set up (but do not solve) and initial value problem describing the amount of salt on each tank.

6. (20 points) Find the general solution of the system

$$\frac{d}{dt} \vec{x}(t) = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \vec{x}(t)$$