Practice Midterm 1
MA 265
Instructor: Javier Zuniga NAME: ______________________________

No calculators or notes allowed. There are a total of 100 points. Read the exam carefully. Show your work for full credit.

1. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix}$$

Compute

(a) $2A - A^T \cdot A$

(b) $\det\left[(3A^T) \cdot (-A) \cdot (A^2)\right]$

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2. (10 points) If $A\vec{x} = \vec{b}$ determine whether the system is inconsistent, has a unique solution or an infinite number of solutions.

(a) $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & -4 & 1 \\ -2 & 8 & -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ and $\vec{b} = \vec{0}$

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3. (10 points) Show that if A is a square matrix then

(a) $A + A^T$ is symmetric

(b) $A - A^T$ is skew-symmetric

(c) if A is skew-symmetric then A^T is also skew-symmetric.

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4. (10 points) Find the adjoint of

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

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5. (20 points) True or False?

(a) The set of twice differentiable real functions is a vector space.

(b) The set of continuous functions is a subspace of the vector space of differentiable functions.

(c) The set of m by n matrices whose sum of entries on the first column adds up to zero is a subspace of $M_{m,n}$.

(d) The set of m by n matrices whose sum of entries on the first row is equals to 1 is a subspace of $M_{m,n}$.

(e) The set of positive real numbers with sum defined as $x \oplus y = x \cdot y$ and $c \odot x = x^c$ is a subspace of $\mathbb{R}^1$.

(f) The subset of $\mathbb{R}^n$ with last entry equal to zero is a subspace.

(g) The line passing through the points $(1, 1)$ and $(1, 0)$ is a subspace of $\mathbb{R}^2$.

(h) The set of matrices with zero determinant is a subspace of $M_{2,2}$.

(i) The plane in $\mathbb{R}^3$ with equation $2x + 3y - z = 0$ is a subspace of $\mathbb{R}^3$.

(j) The set of solutions to an non-homogeneous system is a subspace.

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6. (10 points) Do the polynomials $t^3 + 2t + 1$, $t^2 - t + 2$, $t^3 + 2$, $-t^3 + t^2 - 5t + 2$, $t + 1$ span P_3?

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7. (20 points) Show that the set of 2 by 2 matrices whose sum of all entries is equal to zero is a subspace of $M_{2,2}$. Find a set of matrices that spans this subspace and that is linearly independent. Can you find another set of spanning matrices that is a subset of the given set of matrices?

$$\left\{ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 6 & 4 \\ -7 & -3 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \right\}$$

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8. (10 points) Find a set of vectors spanning the solution space of $A\vec{x} = \vec{0}$ where

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$$

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1	2	3	4	5	6	7	Total