Practice Midterm 2
MA 265
Instructor: Javier Zuniga NAME: ______________________________

No calculators or notes allowed. There are a total of 100 points. Read the exam carefully. Show your work for full credit.

1. (10 points) For which values of α is the set

$$S = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ \alpha - 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3\alpha \\ 4 \\ 2 \\ \alpha \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 4 \end{bmatrix} \right\}$$

linearly independent?

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2. (10 points) Find the dimension of the following subspaces

(a) All 2 by 2 matrices of the form $\begin{bmatrix} a+b & c+d \\ d+e & a+b \end{bmatrix}$.

(b) All polynomials of the form $at^2 + (a+b)t + (a+b+c+d)$.

(c) All vectors of the forms $\begin{bmatrix} a+c \\ a-b \\ b+c \\ -a+b \end{bmatrix}$

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3. (10 points) Show that if V is a vector space and $(,)$ is an inner product then

(a) $(\vec{u}, \vec{0}) = 0$, for any vector $\vec{u}$ in V.

(b) $||c \cdot \vec{u}|| = |c| \cdot ||\vec{u}||$ for any constant $c \in \mathbb{R}$.

(c) $||\vec{u} + \vec{v}|| \leq ||\vec{u}|| + ||\vec{v}||$ for any couple of vectors.

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4. (10 points) Find the rank of the following matrices

(a) $\begin{bmatrix} 1 & -1 & 1 & -1 \\ 2 & 0 & 3 & 1 \\ 0 & 4 & -2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 2 & 0 & 3 & 1 & 2 \\ 0 & 4 & -2 & 0 & -1 \end{bmatrix}$

(c) Is the system

$$\begin{aligned} a - b + c - d &= 1 \\ 2a + 3c + d &= 2 \\ 4b - 2c &= -1 \end{aligned}$$

consistent?

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5. (20 points) True or False?

(a) A linear system of three equations with four variables can have a unique solution.

(b) A linear system of three equations in two variables is always consistent.

(c) Five vectors in $\mathbb{R}^4$ can not be linearly indenpendent.

(d) $L(x, y, z) = \begin{bmatrix} x+y & x-y \\ 0 & 2z \end{bmatrix}_{2,2}$ is a linear transformation.

(e) $L(x, y, z) = t^2 - (x+y)t + (2z)$ is a linear transformation.

(f) $L(x, y) = (-y, -x)$ is an isometry.

(g) $L(x, y) = (x - y, x + y)$ is an isometry.

(h) If rank $A_{3,4}$=3 then any basis of for Null A has only one vector

(i) If nullity of $A_{6,4}$ is 3 then rank $A = 3$.

(j) If a plane in $\mathbb{R}^4$ is the span of two l.i. vectors, then the orthogonal complement of a plane in $\mathbb{R}^4$ is also a plane.

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6. (10 points) Let W be a subspace of $\mathbb{R}^3$ spanned by $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ k \\ 2 \end{bmatrix}, \begin{bmatrix} k+8 \\ 5 \\ 4 \end{bmatrix}$. Determine the values of k so that $W^\perp$ has dimension zero.

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7. (15 points) Compute the distance from the plane $x + y + z = 1$ to the point (-1,-1,2).

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8. (15 points) Let $L(x, y, z) = (-2z, -2y, -2x + 3z)$. Set A to be the matrix associated to this linear transformation. Find a diagonal matrix D and an orthogonal matrix P such that $P^{-1}AP = D$.

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