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MA265 Section 32
Fall 2014

First midterm review problems

The first midterm will be on October 2nd, in class. It covers all the material in the book up through section 4.3. Most of the problems on the midterm will be closely based on the following problems (the actual midterm will be shorter):

1. Consider the linear system

$$\begin{aligned} -x + 2y + 3z &= 0 \\ 3x - 6y + z &= 2. \end{aligned}$$

   (a) Give the augmented matrix corresponding to this system.
   (b) Put this matrix in reduced row echelon form.
   (c) What are all the solutions to the system?

2. Find the values of a for which the linear system is consistent.

$$\begin{aligned} 3x - y &= a \\ 6x - 2y &= 1. \end{aligned}$$

3. Consider the system of equations

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= a \\ x + 3y + bz &= 1. \end{aligned}$$

   (a) For which values of a and b does it have no solution?
   (b) For which values of a and b does it have a unique solution?
   (c) For which values of a and b does it have infinitely many solutions?

4. What condition on a, b, and c makes the system

$$\begin{aligned} 2x + y &= a \\ 3x + z &= b \\ 12x + 3y + 2z &= c \end{aligned}$$

   consistent?

5. Let A, B, and C be invertible matrices. Solve $BA^{-1}B = C^3B^3$ for A.

6. Let
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$
   (a) Find $\det(A)$.
   (b) Is A invertible? If so, find A^{-1}.
   (c) Give the row reduced echelon form of A.
7. Let A, B, and C be $n \times n$ matrices. Which of the following are always true, and which may be false? If a statement may be false, give an example where it is false.
   (a) $AB = BA$.
   (b) If $AB = BC$, then $A = C$.
   (c) If AB is nonsingular, then A is nonsingular and B is nonsingular.
   (d) $\det(AB) = \det(A)\det(B)$.
   (e) $\det(A + B) = \det(A) + \det(B)$.
8. If
$$\det \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} = 10,$$
   then what is
$$\det \begin{bmatrix} a_1 + 2c_1 & a_3 + 2c_3 & a_2 + 2c_2 & a_4 + 2c_4 \\ b_1 & b_3 & b_2 & b_4 \\ c_1 & c_3 & c_2 & c_4 \\ 3d_1 & 3d_3 & 3d_2 & 3d_4 \end{bmatrix}?$$
9. If A is a 2×2 matrix with $\det A = 5$ and $B = 3A$, what is $\det(B^T A^{-1})$?
10. Let
$$A = \begin{bmatrix} 0 & 2 & 3 & 0 & 0 \\ 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 0 & 3 \end{bmatrix}.$$
   What is the determinant of A?
11. If A is a nonsingular 3×3 matrix and $2A^3 = A$, what are the possible values of $\det(A)$?
12. Let
$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

(a) Find $\det A$.

(b) Use the adjoint method to find the entry in the second row and second column of A^{-1}.

(c) Suppose

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Use Cramer's rule to find y.

13. Suppose the vector $\begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$ is given the tail $P(6, 0, -1)$. What is the head of the resulting directed line segment?

14. Define the following operations on the set of ordered pairs of real numbers (x, y)

$$(x, y) \oplus (x', y') = (x + x', y + y' + 1), \qquad r \odot (x, y) = (rx, ry).$$

(a) Does $(x, y) \oplus (x', y')$ always equal $(x', y') \oplus (x, y)$?

(b) Find an element (x_0, y_0) such that

$$(x, y) \oplus (x_0, y_0) = (x_0, y_0) \oplus (x, y) = (x, y)$$

for any (x, y).

(c) Does $r \odot ((x, y) \oplus (x', y'))$ always equal $(r \odot (x, y)) \oplus (r \odot (x', y'))$?

(d) Is the set of ordered pairs of real numbers a vector space under the above operations $\oplus$ and $\odot$?

15. For what values of a is the matrix $\begin{bmatrix} 3 & a \\ 4 & 8 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$?

16. Which of the following subsets of R_4 are subspaces?

(a) $\{[a\ b\ c\ d]$ such that $a + b = 2\}$

(b) $\{[a\ b\ c\ d]$ such that $a + b = 0\}$

(c) $\{[a\ b\ c\ d]$ such that $a + b + c = 0$ and $2d = 3c - a\}$