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MA265 Section 32
Fall 2014

Second midterm review problems

The second midterm will be on November 6th, in class. It covers all the material in the book up through section 5.6. Most of the problems on the midterm will be closely based on the following problems (the actual midterm will be shorter):

1. Find all real numbers a such that the vectors $[1, 2]$, $[a, 2]$, and $[2, a]$ span R_2.
2. Find all real numbers a such that $3at^2 - a^2t + 2$ is in the span of $2t^2 + t + 3$, $t + 1$, and $3t^2 + t + 4$.
3. Find all real numbers a such that the matrices $\begin{bmatrix} a & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ a & 0 \end{bmatrix}$ linearly independent.
4. Find a basis for the subspace of R_5 spanned by $[5, 1, 4, 3, 2]$, $[0, 0, 1, 0, 0]$, $[0, 0, 0, 0, 2]$, $[10, 2, 5, 6, 4]$, and $[10, 2, 8, 6, 5]$.
5. Let $A = \begin{bmatrix} 3 & 3 & 2 & 5 \\ 0 & 3 & 1 & -2 \\ 1 & 2 & 1 & 1 \end{bmatrix}$.
   (a) Find a basis for the column space of A.
   (b) Find a basis for the null space of A.
   (c) What is the rank of A?
   (d) What is the nullity of A?
6. Suppose the matrix $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & p & q \end{bmatrix}$ has reduced row echelon form $\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
   (a) Find the determinant of A.
   (b) Find the rank of A.

(c) Find the nullity of A.

(d) Find a basis for the column space of A.

(e) Find a basis for the row space of A.

(f) Find a basis for the null space of A.

7. Let A be a 4×6 matrix whose nullspace is spanned by $\begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -2 \\ 3 \\ -1 \\ -1 \\ 2 \end{bmatrix}$.

(a) Find the rank of A.

(b) Is $A \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 0$? Explain your answer.

(c) How many solutions are there to the equation $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$? Explain your answer.

8. Find a basis for the set of 2×2 matrices of the form $\begin{bmatrix} a+b+3c & -2a+b \\ 3a+b+5c & 0 \end{bmatrix}$, where a, b and c are any real numbers.

9. Find a basis for the set of 3×2 matrices of the form $\begin{bmatrix} a+2c & b+c \\ a+b+3c & a+b+3c \\ 2a+b+5c & 3a+6c \end{bmatrix}$, where a, b and c are any real numbers.

10. Find all real numbers a such that the vectors $v_1 = \begin{bmatrix} -8 \\ a \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ a \\ 5 \end{bmatrix}$ are orthogonal.

11. Let x and y be two vectors in R^3 satisfying $x \cdot y = 0$, $\|x\| = 2$, and $\|y\| = 3$. What is $\|2x - 3y\|$?

12. Find the orthonormal set of vectors $\{w_1, w_2, w_3\}$ obtained by applying the Gram–Schmidt process to the vectors $u_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$,

$u_3 = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$.

13. Let W be the subspace of R_3 spanned by $[1, 2, 3]$, $[2, k, 3]$, $[4, 5, k]$. Find all values of k such that the dimension of $W^\perp$ is 0.

14. Let W be the subspace of R^4 spanned by $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 10 \\ 11 \\ 12 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. Find a basis for the orthogonal complement $W^\perp$

15. Let W be the subspace of R_4 spanned by the orthonormal set $\{[1/\sqrt{2}, 0, 0, 1/\sqrt{2}]$, $[0, 0, 1, 0]$, $[-1/\sqrt{2}, 0, 0, 1/\sqrt{2}]\}$. Suppose $[2, 1, 0, 4] = w + v$ with w in W and v in $W^\perp$. Find v and w.

16. Let $w_1 = \begin{bmatrix} 0 \\ -24 \\ 0 \end{bmatrix}$, $w_2 = \begin{bmatrix} 7 \\ 0 \\ 6 \end{bmatrix}$, $w_3 = \begin{bmatrix} -6 \\ 0 \\ 7 \end{bmatrix}$ be an orthogonal set that spans R^3. Let $v = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$. If $v = a_1w_1 + a_2w_2 + a_3w_3$, what are a_1, a_2, and a_3?

17. Let W be the subspace of R_3 with basis $\{[-1, 1, 0], [0, 1, 1]\}$. Find the vector in W closest to $[2, 4, 0]$.

18. Find the least squares solution to

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

19. Find the least squares line $y = ax + b$ for the data table below:

x	-3	0	1	2
y	1	0	1	2