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MA266 Section 53
Spring 2015

Second midterm review problems

The second midterm will be on March 31st, in class. It covers all the material in the book up through section 6.1. Most of the problems on the midterm will be closely based on the following problems (the actual midterm will be shorter):

1. Let a, b, and c be real numbers. For which values of a, b, and c is the following differential equation linear
$$at^2y'' + y' + byy' + cy = 0?$$
2. Solve the initial value problem
$$y'' + 9y = 18e^{2t} + 9\cos(3t), \qquad y(0) = 0,\ y'(0) = 0.$$
3. Find the largest open interval in which a solution to the initial value problem
$$(t-5)^2y'' = (t-3)y' + y\ln(t-2) + \cos(t), \qquad y(4) = 1,\ y'(4) = 0,$$
is guaranteed to exist. Do not try to find the solution.
4. Given that $y_1(t) = t^{-2}$ is a particular solution to
$$t^2y'' + 5ty' + 4y = 0, \qquad t > 0,$$
find the general solution to the differential equation.
5. A mass weighing 2 lb, when hung from the ceiling, stretches a spring by 9 in. The mass is attached to a damper with a damping constant of 10 lb sec/ft. Suppose it is displaced upward from the equilibrium position by 2 in, and set in motion with an initial velocity of 2 in/sec directed upward. Let $u(t)$ be the displacement in feet after t seconds, with the downward direction taken as positive. What is the initial value problem satisfied by $u(t)$? Do not solve the initial value problem. (Hint: the acceleration of gravity is 32 ft/sec, and there are 12 inches in 1 foot).
6. Find the general solution to
$$2y'' + 6y' + 10y = 0.$$
7. Find the largest open interval in which a solution to the initial value problem
$$\cos(t)y^{(4)} - e^{\sin(t)}y''' = (t-1)y'' + y\ln(t-2) + \sin(t),$$

$$y(0) = 1,\ y'(0) = 1,\ y''(0) = 2,\ y'''(0) = 3$$

is guaranteed to exist. Do not try to find the solution.

8. Find the Laplace transform of

$$f(t) = \begin{cases} 0, & 0 \le t < 1, \\ t, & 1 \le t < 2, \\ 0, & 2 \le t < \infty. \end{cases}$$

9. According to the method of undetermined coefficients, what is the proper form of a particular solution to the differential equation

$$y^{(4)} - 3y''' = 8t^2 + 3 + te^t?$$

Do not determine the coefficients.

10. Find a particular solution to the equation

$$y'' - 2y' + y = \frac{e^t}{1+t}, \qquad t > -1.$$

11. Find the general solution to the differential equation

$$y^{(4)} - 81y = 0.$$

12. Find the general solution to the differential equation.

$$y^{(4)} - y = e^t.$$

13. A mass of 2 kg hangs from a spring with a spring constant of 18 kg/sec sec. Suppose you pull the mass downward an additional 3 cm from its equilibrium position, and then release it with a downward initial velocity of 4 cm/sec. If there is no damping, what are the amplitude and period of the resulting oscillatory motion?

14. Find the Laplace transform of $f(t) = te^{-t}$.

15. Suppose the position of a certain mass-spring system satisfies the initial value problem

$$u'' + \gamma u' + 4u = 0, \qquad u(0) = 2,\ u'(0) = u_0,$$

where $\gamma > 0$ is a damping coefficient and u_0 is a real number. For values of γ is the motion critically damped? Find the solution in terms of u_0 in that case. How many times does the system pass through the equilibrium position for each value of u_0?