MA 341 final review problems

Version as of 4/13.

The final is on Friday, 5/8, from 8 to 9 pm in WTHR 104. No notes, books, or electronic devices allowed. Most of the exam will be closely based on problems from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. The problems from the midterm reviews and exams.

2. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a continuous function such that

$$\lim_{x\to\infty} f(x) = 37, \qquad \lim_{x\to-\infty} f(x) = -\infty.$$

   Prove that f is bounded above.

3. Let $f_n \to f$ uniformly on an interval (a, b). Let L_1, L_2, ... be a sequence of real numbers converging to a real number L. Prove that if $\lim_{x\to a} f_n(x) = L_n$, then $\lim_{x\to a} f(x) = L$.

4. Let

$$f(x) = \sum_{n=0}^{\infty} \frac{\cos(nx)}{1 + 2^n + 4^n}.$$

   Prove that f is continuous and find

$$\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(x)\,dx.$$

5. Evaluate the seventy-seventh derivative at $x = 0$ of each of the following functions:

$$\frac{x^5}{1-x^3}, \qquad \frac{x^7}{(1-x^2)^2}, \qquad \sin(x^7 + x^{100})$$

6. More problems to be added soon.

Here are some hints.

1.
2. See problem 8 from the first midterm review.
3. See Theorem 2.7 and Exercise 2.8 of the chapter on Sequences of Functions.
4. $1/3$.
5. $77!$, $36 \cdot 77!$, $-77!/11!$