MA 341 first midterm review problems

Version as of 2/3.

The first midterm is on Monday, 2/16, from 8 to 9 pm in ME 1130. No notes, books, or electronic devices allowed. Most of the exam will be closely based on problems from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. For each of the following sequences, find the limit L. Then find N such that $|s_n - L| < 0.1$ for $n > N$.

   (a) $s_n = 10n^{-1/3}$, (b) $s_n = \sqrt{n+100} - \sqrt{n}$, (c) $s_n = \dfrac{2n^3 + \cos n + 2^{-n}}{n^3+4}$.

2. Repeat the above, but find N which is within a factor of 10 of the best possible.

3. Find a natural number N such that if $|a-5| < \varepsilon < 1$ then $|a^2 - 25| < N\varepsilon$.

4. Find a number c such that

   $$\text{if } \left|a - \frac{1}{8}\right| < \varepsilon < \frac{1}{16}, \qquad \text{then } \left|\frac{1}{\sqrt{a}} - \sqrt{8}\right| < c\varepsilon.$$

5. Let a, b and ε be real numbers such that $|a-3| < \varepsilon$, $|a-b| < 2\varepsilon$, and $|b - 3.5| < \varepsilon$. What are the possible values of ε?

6. Prove that $\sqrt{k+1} - \sqrt{k} < \frac{1}{2\sqrt{k}} < \sqrt{k} - \sqrt{k-1}$ for any positive integer k, and use this to show that

   $$2\sqrt{n+1} - 2 < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} < 2\sqrt{n}$$

   and hence that

   $$\frac{2}{\sqrt{n} + \sqrt{n+1}} - 2 < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} - 2\sqrt{n} < 0$$

7. Prove that the recursive sequence $s_{n+1} = \sqrt{2s_n}$ is montone and bounded, and find the limit, in the cases

   (a) $s_1 = 1$.

   (b) $s_1 = 10$.

8. More problems to be added soon.

Here are some hints.

1. (a) $L = 0,\ N = 10^6$
   (b) $L = 0,\ N = 2.5 \cdot 10^5$
   (c) $L = 2,\ N = 5$
2. (a) $s_{10^5} = 10^{-2/3} > 10^{-1}$
   (b) $s_{2.5 \cdot 10^5 - 100} > 0.1$
   (c) $s_0 = 5/2 > 2.1$
3. 6
4. $\dfrac{32}{1 + \sqrt{1/2}}$
5. ε can be any number greater than 1/8.
6. Use that $0 < a < b$ and $0 < c \leq d$ implies $a + c < b + d$. and that $a < b$ and $c \leq d$ implies $a + c < b + d$.
7. In the two cases the monotonicty is opposite but the limit is always 2.