MA 341 second midterm review problems

Version as of 3/12.

The second midterm is on Monday, 4/6, from 8 to 9 pm in ME 1130. No notes, books, or electronic devices allowed. Most of the exam will be closely based on problems from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. Determine if each of the following statements is true or false. If true, give a proof. If false, give a counterexample.

   (a) If $E \subset \mathbb{R}$ is bounded and nonempty, and $\alpha = \sup E$, then for every $\varepsilon > 0$ there is $x \in E$ such that $x \in [\alpha - \varepsilon, \alpha]$.

   (b) If $E \subset \mathbb{R}$ is bounded and nonempty, and $\alpha = \sup E$, then for every $\varepsilon > 0$ there is $x \in E$ such that $x \in (\alpha - \varepsilon, \alpha)$.

2. Evaluate
$$\frac{d}{dx} \int_x^{x^2} \sin(t^2)\, dt$$

3. Find numbers a and b such that for all real c we have
$$0 < a \leq \int_0^2 \frac{x^2}{4 + \sin(cx)} dx \leq b < 1.$$

4. Find a number $c < 1$ such that for all real a we have
$$\int_0^{10} \frac{\cos(ax)}{1 + e^x}\, dx \leq c$$

5. In each of the following problems, find $\delta > 0$ such that $|f(x) - f(x_0)| < 0.1$ when $|x - x_0| < \delta$.

   (a)
$$f(x) = \frac{2}{1 + 3x^2}, \qquad x_0 = 0$$

   (b)
$$f(x) = \int_0^2 \frac{3}{1 + 4|x|t^4}\, dt, \qquad x_0 = 0$$

   (c)
$$f(x) = \frac{100}{9 + x}, \qquad x_0 = 1$$

6. Suppose that $f'(x) \geq 10$ for all x in $(0, 5)$. Find $a > 0$, as large as possible, such that there is guaranteed to be an interval I of length a contained in $(0, 5)$ such that $|f| \geq 2$ on I. Justify your answer using the mean value theorem.

7. More problems to be added soon.

Here are some hints.

1. (a) This is true, prove it by proving the contrapositive.
   (b) This is false, show that any set with just one element is a counterexample.
2. $2x\sin(x^4) - \sin(x^2)$.
3. Use $-1 \leq \sin \leq 1$.
4. The integrand is bounded above by e^{-x}.
5. $\sqrt{1/60}$, $1/768$, 0.08
6. Start with the case $f(x) = 10x + b$, where b is a constant.