MA 351 final review problems

Version as of May 2nd.

The final will be as scheduled here. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. All of the problems from both midterm reviews.

2. Let
$$A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ -1 & -2 & 1 & 5 \\ 3 & 6 & 2 & 0 \end{bmatrix}.$$
   (a) Find the rref, rank, nullity, a basis for the range, and a basis for the nullspace.

   (b) Find a basis for the range that has all integer entries and as many zero entries as possible.

   (c) Find an orthonormal basis for the nullspace.

3. Let $\vec{v}$ and $\vec{w}$ be vectors wuch that $\|\vec{v}\| = 2$, $\|2\vec{v} - 3\vec{w}\| = 6$, and $\vec{v} \cdot \vec{w} = 0$. Find $\|\vec{w}\|$.

4. Let S_1 be the span of $\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right)$ and let S_2 be the span of $\left(\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \\ 7 \end{bmatrix} \right)$. Find a basis for the space of vectors that are in both S_1 and S_2.

5. Problems 2–5 from Section 6.4 of Shifrin and Adams.

6. Find the least squares solution to $\begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$.

7. Consider the problem of fitting the parabola $y = ax^2 + bx + c$ to the points $(-1, 0)$, $(0, 1)$, $(1, 0)$, and $(2, 0)$. Find a matrix A and a vector $\vec{w}$ such that the unique solution to $A \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \vec{w}$ is the least square solution to the problem.

Answer key

1.

2. (a) $\begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, 2, 2, $\left(\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right)$, and $\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right)$.

   (b) $\left(\begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} \right)$.

   (c) $\left(\frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{3\sqrt{30}} \begin{bmatrix} 2 \\ 4 \\ -15 \\ 5 \end{bmatrix} \right)$.

3. $\frac{2}{3}\sqrt{5}$.

4. $\left(\begin{bmatrix} 3 \\ 8 \\ 5 \end{bmatrix} \right)$

5.

   6.4.2. $\begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

   6.4.3. $\frac{1}{3} \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix}$

   6.4.4. $\frac{1}{2} \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$

   6.4.5. No (find all eigenvectors), yes (symmetric), yes (all eigenvalues different), yes (find all eigenvectors).

6. $x_1 = 19/3$, $x_2 = -2$.

7. $A = \begin{bmatrix} 18 & 8 & 6 \\ 8 & 6 & 2 \\ 6 & 2 & 4 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.