**MA 351 second midterm review problems**
Version as of March 15th.

The second midterm will be in class on Monday, March 31st. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. Find a basis $\vec{v}_1$, $\vec{v}_2$ that diagonalizes the filter with matrix $\begin{bmatrix} 5/2 & 1/2 \\ 2 & 1 \end{bmatrix}$.

2. For which real values of $a$ is the matrix $\begin{bmatrix} 3 & a^2 - a \\ a & 3 \end{bmatrix}$ diagonalizable using real numbers? For which values are complex numbers needed? For which values is the matrix not diagonalizable? Mark the different ranges of values on a number line.

3. For which real values of $a$ is the matrix $\begin{bmatrix} -3 & a-4 \\ a & 1 \end{bmatrix}$ invertible?

4. Consider the recursive sequence $x_{n+2} = x_{n+1} + 6x_n$, where $x_0 = 1$, $x_1 = 2$. Find $x_{10}$ in terms of $2^{10}$ and $3^{10}$.

5. For which values of $\alpha$ is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & \alpha & 0 \\ 3 & 0 & \alpha \end{bmatrix}$ invertible? Find the inverse for those values.

6. Find the determinants of the following matrices:

$$\begin{bmatrix} 1 & 2 & 1 \\ 4 & 0 & -4 \\ 2 & -1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 0 & 4 & -2 & 3 \\ 0 & 0 & 0 & -\frac{3}{4} \\ 2 & 5 & -1 & 0 \\ 0 & 0 & \frac{2}{3} & \pi \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 7 & 0 & 0 & 0 & 6 \\ 8 & 0 & 0 & 0 & 8 \\ 6 & 0 & 0 & 0 & 7 \\ 5 & 4 & 3 & 2 & 1 \end{bmatrix}.$$

7. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix}$. Find matrices $V$, $D$, and $V^{-1}$ such that $D$ is diagonal and $A = VDV^{-1}$.

8. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \pi & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Find matrices $V$, $D$, and $V^{-1}$ such that $D$ is diagonal and $A = VDV^{-1}$.

9. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix}$. Find matrices $V$ and $D$ such that $D$ is diagonal and $A = VDV^{-1}$. (For this one, omit finding $V^{-1}$.)

10. For which values of $\alpha$ are the vectors $\begin{bmatrix} 1 \\ -3 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -4 \\ \pi \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ \alpha \end{bmatrix}$ independent?

Here are short answers; a proper solution includes some clear steps leading to these answers.

1. One possibility is $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$.
2. Real numbers work when $a > 1$ or $a = 0$. Complex numbers are needed when $a < 1$ and $a \neq 0$. The matrix is not diagonalizable when $a = 1$.
3. All values other than $a = 1$ and $a = 3$.
4. $x_{10} = (2^{10} + 4 \cdot 3^{10})/5$.
5. For all values other than $\alpha = 0$ and $\alpha = 6$. The inverse is $\begin{bmatrix} \frac{\alpha}{\alpha-6} & 0 & \frac{2}{6-\alpha} \\ 0 & \frac{1}{\alpha} & 0 \\ \frac{3}{6-\alpha} & 0 & \frac{1}{\alpha-6} \end{bmatrix}$.
6. $-40$, 4, 0.
7. One possibility is $V = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, $V^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$.
8. One possibility is $V = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \pi \end{bmatrix}$, $V^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$.
9. One possibility is $V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & -1 & 0 & 3 \\ -1 & 0 & 0 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$.
10. For $\alpha \neq (1 + 2\pi)/3$.