MA 362
Spring 2017

Homework 3

Due February 2nd in class or by 3:20 pm in MATH 602.

1. Let
$$f(x,y) = \begin{cases} 0, & \text{if } (x,y) = (0,0), \\ \dfrac{y^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0). \end{cases}$$
   (a) Is f continuous at all points (x,y) in $\mathbb{R}^2$?

   (b) Is f differentiable at all points (x,y) in $\mathbb{R}^2$?

   (c) Is f C^1 at all points (x,y) in $\mathbb{R}^2$?

   Explain your answer in each case.

2. (a) Let $z = \sin(x-y)$. Use the chain rule to evaluate
$$\partial_x z + \partial_y z.$$
   (b) Let $z = f(ax+by)$, where a and b are given constants, and f is a given differentiable function. Use the chain rule to find all constants c and d such that
$$c\partial_x z + d\partial_y z = 0.$$

3. Consider the equation
$$\sin(xyz) + x + y^2 + z^3 = 0.$$
   (a) Is there a differentiable function f such that $x = f(y,z)$ solves the equation near $(0,0,0)$? If so, find $\partial_y f(0,0)$ and $\partial_z f(0,0)$.

   (b) Is there a differentiable function g such that $y = f(x,z)$ solves the equation near $(0,0,0)$? If so, find $\partial_x g(0,0)$ and $\partial_z g(0,0)$.

   (c) Is there a differentiable function h such that $z = h(x,y)$ solves the equation near $(0,0,0)$? If so, find $\partial_x h(0,0)$ and $\partial_y h(0,0)$.