MA 362
Spring 2017

# Homework 5

Due February 23rd in class or by 3:20 pm in MATH 602.

1. Which of the following differential 1 forms can be written as $df$ for some function $f\colon \mathbb{R}^2 \to \mathbb{R}$? If the answer is yes, find such a function $f$.

   (a) $x^2dx - y^2dy$,

   (b) $y^2dx - x^2dy$,

   (c) $\sin x \sin y\, dx + \cos x \cos y\, dy$,

   (d) $\sin x \sin y\, dx - \cos x \cos y\, dy$.

2. (a) Find a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ such that
   $$\nabla f = (4xe^{2x^2+y^2}, 2ye^{2x^2+y^2}).$$

   (b) Sketch some level sets of $f$, and use them to sketch the vector field $(4xe^{2x^2+y^2}, 2ye^{2x^2+y^2})$.

3. (a) Find a function $f$ such that
   $$\nabla f = \left(\frac{1}{2} - \frac{y^2}{2x^2}, \frac{y}{x}\right),$$
   when $x > 0$.

   (b) Sketch some level sets of $f$, and use them to sketch the vector field
   $$\left(\frac{1}{2} - \frac{y^2}{2x^2}, \frac{y}{x}\right),$$
   in the region where $x > 0$.

   *Hint:* Completing the square will simplify the formula for the level sets.