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MA 366
Spring 2019

Homework 10

Due April 9th at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. For each of the following systems, find the equilibria and decide if each is stable or unstable.

   (a)

   $$x_1' = x_2^3 - x_1^3,$$
   $$x_2' = x_1^2 + x_2^2 - 2.$$

   (b)

   $$x_1' = \sin(\pi x_2),$$
   $$x_2' = x_1^2 + x_2^2 - 1000.$$

2. Consider the system

   $$x_1' = x_1(1 - x_1),$$
   $$x_2' = x_2(2 - x_2 - ax_1),$$

   where $a \geq 0$ is given. This models two populations where the first hinders the second but not vice versa. In this problem x_1 and x_2 are always nonnegative.

   (a) Find all the equilibrium solutions.

   (b) For which values of a is there an equilbrium solution with both populations positive?

   (c) For each equilibrium solution, find the eigenvalues of the matrix of the linear approximation.

   (d) For which values of a does the linear approximation give enough information to classify all the equilibria as stable or unstable? Classify the equilibria in those cases.

   (e) Draw a phase portrait near each equilibrium in the case $a = 0$.

   (f) Draw a phase portrait near each equilibrium in the case $a = 1$.

   (g) Draw a phase portrait near each equilibrium in the case $a = 3$.