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MA 366
Spring 2019

Homework 11

Due April 23rd at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. For each of the following systems, find $a > 0$ and $b > 0$ such that $L(x, y) = ax^2 + by^2$ obeys $\frac{d}{dt}L \neq 0$ whenever $(x, y) \neq (0, 0)$. (This makes L a Liapounov function.) State whether the origin is a stable or unstable equilibrium in each case.

   (a)

   $$\begin{aligned} x' &= -x^3 + 7xy^2, \\ y' &= -3x^2y - y^3. \end{aligned}$$

   (b)

   $$\begin{aligned} x' &= x^3 - y^3, \\ y' &= 3xy^2 + 4x^2y + 5y^3. \end{aligned}$$

2. Use the Laplace transform to solve

   $$y'' - y' - 6y = 100e^{8t}, \qquad y(0) = 5, \qquad y'(0) = 20.$$

3. Use the Laplace transform to find all real numbers a and b such that the solution to

   $$y'' - y' - 6y = 3e^{-4t} + 5e^{-6t}, \qquad y(0) = a, \qquad y'(0) = b,$$

   obeys $\lim_{t\to\infty} y(t) = 0$. (You do not have to fully find $y(t)$.)

4. The motion of a forced spring mass system is given by

   $$y''(t) + y(t) = \begin{cases} 1, & t < 1, \\ 0, & t \geq 1, \end{cases} \qquad y(0) = 1, \quad y'(0) = 0.$$

   (a) Use the Laplace transform to find $y(t)$.

   (b) Find the maximum of $y(t)$. At what times is the maximum achieved?

   (c) Find the minimum of $y(t)$. At what times is the minimum achieved?