

Homework 5

Due February 19th at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. Exercise 2.2.8 from page 72 of
<https://www.jirka.org/diffyqs/diffyqs.pdf>
2. Exercise 2.2.9 from page 72 of
<https://www.jirka.org/diffyqs/diffyqs.pdf>
3. Exercise 2.2.11 from page 72 of
<https://www.jirka.org/diffyqs/diffyqs.pdf>
4. Exercise 2.2.16 from page 73 of
<https://www.jirka.org/diffyqs/diffyqs.pdf>
5. Consider the differential equation

$$y''(t) + p(t)y'(t) + q(t)y(t) = 0,$$

where p and q are given functions. Show that $y(t) = \sin(t^2)$ is not a solution to this equation in any interval containing $t = 0$, regardless of what p and q are.

Hint: Plug that choice of y into the equation and get a contradiction when $t = 0$.

6. In the last homework you found the general solution to *Legendre's equation*:

$$(1 - x^2)y''(x) - 2xy'(x) + n(n + 1)y(x) = 0.$$

when $n = 1$, for $-1 < x < 1$, using the method of reduction of order. Now do the same problem a different way using these steps:

- (a) Let $y_1(x) = x^m$, where m has the value you found in the last homework. Let $W(x) = y_1(x)y'(x) - y_1'(x)y(x)$ be the Wronskian of this function and the general solution you are looking for. Find a polynomial $P(x)$ such that

$$W(x) = \frac{C}{P(x)},$$

where C is an unknown constant.

- (b) Plug in $W(x) = y_1(x)y'(x) - y_1'(x)y(x)$ into the formula $W(x) = C/P(x)$ and solve for y . Express your answer using one natural logarithm, one square root, and no absolute values. You may use any of the partial fraction decompositions from last week that you need.