

Homework 6

Due February 26th at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. Exercise 2.5.2 from page 93 of
<https://www.jirka.org/diffyqs/diffyqs.pdf>
2. Consider the differential equation

$$y'' + 2y' + y = t + e^t.$$

- (a) Find the general solution to the equation.
 - (b) Solve the initial value problem with $y(0) = 0$ and $y'(0) = 0$.
3. Consider the differential equation

$$y'' + 4y' + 6y = te^t + t^2 + \cos(3t).$$

What is a good form for the initial guess if the method of undetermined coefficients is to be used? (You do not have to find the coefficients).

4. Exercise 2.4.4 from page 86 of
<https://www.jirka.org/diffyqs/diffyqs.pdf>
5. An undamped spring-mass system oscillates with frequency 5 s^{-1} , amplitude 10 cm, and phase $7\pi/6$. Find the period, the starting position, and the starting velocity.
6. A mass stretches a spring by 10 cm. The mass is released from the equilibrium position with an upwards starting velocity of $v \text{ cm/s}$, where $v > 0$ is a given constant. There is no damping. In this problem, for simplicity use $g = 1000 \text{ cm/s}^2$.
 - (a) Find the frequency, period, amplitude, and phase of the resulting periodic motion, in terms of v . Include units your answers.
 - (b) Suppose that we don't want the mass to ever be more than 5 cm below the equilibrium position. What values of v may we use?
 - (c) Suppose we want the mass to be 5 cm below the equilibrium position after 10 seconds. What values of v may we use? What is the smallest value of v that works?

(Of course, if v is big enough, eventually the model breaks down, the string is destroyed, etc. but for this problem ignore these issues)