

Homework 7

Due March 5th at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. A damped spring-mass system is governed by the initial value problem

$$u'' + \sqrt{2}u' + u = 0, \quad u(0) = 0, \quad u'(0) = -7.$$

- (a) Find $u(t)$.
 - (b) What are the quasiperiod and quasifrequency?
 - (c) What is the minimum value of $u(t)$ for $t > 0$?
 - (d) What is the maximum value of $u(t)$ for $t > 0$?
 - (e) For which values of $t > 0$ is $u(t) = 0$? Which value is closest to $t = 10$?
 - (f) Find all $t > 0$ such that u has a local maximum at t .
 - (g) If $t_1 < t_2$ are two successive local maximum points for u , what is $u(t_2)/u(t_1)$?
2. A forced spring-mass system is governed by the initial value problem

$$u'' + 9u = \cos(\omega t) + 2\sin(\omega t), \quad u(0) = 4, \quad u'(0) = 5,$$

where $\omega > 0$ is given. Find $u(t)$.

3. A damped and forced spring-mass system is governed by the initial value problem

$$u'' + 3u' + 9u = \cos(\omega t) + 2\sin(\omega t), \quad u(0) = 4.3, \quad u'(0) = 5.04,$$

where $\omega > 0$ is given.

- (a) Find the amplitude of the steady state solution. (You do not have to find $u(t)$).
- (b) What value of ω maximizes the answer to part (a) above?