

Homework 8

Due March 19th at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. Find the solution to the initial value problem

$$y''' + 4y'' - y' - 22y = 0, \quad y(0) = 1, \quad y'(0) = y''(0) = 0.$$

Hint: e^{2t} is a particular solution to the differential equation.

2. Find the general solution to the equation

$$y^{(4)} - 16y = 32t^4 + 32.$$

3. Consider the system

$$\begin{aligned} x_1' &= -2x_1 + 2x_2, \\ x_2' &= -2x_1 + 3x_2. \end{aligned}$$

- (a) Find the solution to the initial value problem $x_1(0) = 0$, $x_2(0) = 1$.
- (b) Sketch a phase portrait of the system. Identify all equilibrium and straight line solutions.
- (c) Is the equilibrium stable? Is it a node, a saddle, or neither?

4. Consider the system

$$\begin{aligned} x_1' &= x_1 + 2x_2, \\ x_2' &= 4x_1 - x_2. \end{aligned}$$

- (a) Sketch a phase portrait of the system. Identify all equilibrium and straight line solutions.
- (b) Is the equilibrium stable? Is it a node, a saddle, or neither?
- (c) Consider the solution to the initial value problem $x_1(0) = 1$, $x_2(0) = a$, where a is a given real number. Find $\lim_{t \rightarrow \infty} x_1(t)$.