

Homework 9

Due April 2nd at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

In this homework assignment you are asked to analyze two linearizations of the nonlinear system

$$\begin{aligned}x'_1 &= x_2, \\x'_2 &= -5 \sin x_1 - 2x_2,\end{aligned}$$

which represents the motion of a damped pendulum.

1. The linear system

$$\begin{aligned}x'_1 &= x_2, \\x'_2 &= 5x_1 - 2x_2 - 5\pi,\end{aligned}$$

gives good approximations to the nonlinear system near $(\pi, 0)$.

- (a) Sketch a phase portrait of this linear system. Identify all equilibrium and straight line solutions.
- (b) Is the equilibrium stable?
- (c) Suppose $x_1(0) = 1$ and $x_2(0) = a$ for some real a , and

$$\lim_{t \rightarrow \infty} x_1(t) = \pi, \quad \lim_{t \rightarrow \infty} x_2(t) = 0.$$

What is a ?

2. The linear system

$$\begin{aligned}x'_1 &= x_2, \\x'_2 &= -5x_1 - 2x_2,\end{aligned}$$

gives good approximations to the nonlinear system near $(0, 0)$.

- (a) Sketch a phase portrait of this linear system. Identify all equilibrium and straight line solutions.
- (b) Is the equilibrium stable?
- (c) If $x_1(0) = x_2(0) = 1$, find the smallest $t > 0$ such that $x_1(t) = 0$.