

## Homework 6

Due March 10th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake.

**NOTE:** For this homework, each time you find a Fourier series, write out the first few terms without using any complex numbers, (enough terms so that the pattern is clear) and simplify them as much as possible.

1. Let  $f(x) = x$  for  $0 < x < 1$ .
  - (a) Let  $f_1$  be the odd extension of  $f$  with period 2. Sketch  $f_1$  and find its Fourier series.
  - (b) Let  $f_2$  be the even extension of  $f$  with period 2. Sketch  $f_2$  and find its Fourier series.
  - (c) Let  $f_3$  be the extension of  $f$  with period 1. Sketch  $f_3$  and find its Fourier series.
  - (d) In parts (a), (b), and (c) above the functions have been left undefined when  $x$  is an integer. What should the values be at those points so that the Fourier series converge everywhere?
  - (e) \*This part is not to be handed in but is just for fun.\* Notice how the speed at which the terms go to zero is related to the sizes of the jumps. You may enjoy taking various partial sums  $S_N(x)$  and plotting  $x - S_N(x)$  on a computer (for instance on Desmos) to compare how well they do. The difference is significant even on intervals like  $.4 \leq x \leq .6$  that stay well away from the endpoints.
2. (a) Let  $a > 0$ , let  $L > b > 0$ , and let

$$f(x) = \begin{cases} ax/b, & \text{for } 0 \leq x \leq b, \\ a(L-x)/(L-b), & \text{for } b \leq x \leq L. \end{cases}$$

Sketch the graph of  $f$  on  $[0, L]$  and find its Fourier sine series.

- (b) For which values of  $b/L$  does the first term vanish?
- (c) For which values of  $b/L$  does the second term vanish?
- (d) For which values of  $b/L$  does the third term vanish?
- (e) Let  $k$  be a positive integer. For how many values of  $b/L$  does the  $k$ -th term vanish? What are those values?
- (f) If the  $k$ -th term vanishes, which other terms must also vanish?