Final Exam

Due Wednesday, May 6th by 12:30 pm by email. You may look up any information you need, but you may not use any calculating devices or programs or discuss the problems with anyone but me. Justify your answers. Please let me know if you have a question or find a mistake.

1. Suppose that $f\colon \mathbb{R}^3 \to \mathbb{R}$ is differentiable and satisfies

$$\nabla f(0,1,1) = (1,2,3),$$

and that $z = z(x,y)$ obeys the implicit equation

$$e^{xy} + e^{yz} + e^{xz} = 2 + e,$$

where x and y are independent variables. Let $g(x,y) = f(x,y,z(x,y))$.

   (a) What is $\nabla z(0,1)$?

   (b) What is $\nabla g(0,1)$?

2. Let C be the line segment from $(-1,-1)$ to $(1,1)$. Evaluate the following line integrals

   (a)
$$\int_C x^2 ds,$$

   (b)
$$\int_C \sin^3(y^7)ds,$$

   (c)
$$\int_C xydx + xydy,$$

   (d)
$$\int_C \left(xy + \cos^2(x^3)\sin^2(y^3)\right) dx + \left(xy - \cos^2(x^3)\sin^2(y^3)\right) dy.$$

3. Evaluate
$$\int_C ydx - xdy,$$

   where C is the oriented curve consisting of the line segment from $(2,0)$ to $(-2,0)$ followed by the top half of the circle $x^2 + y^2 = 4$ from $(-2,0)$ back to $(2,0)$.

4. Let S be the surface given by $x^2 + y^2 = 1$, $x \geq 0$, $-1 \leq z \leq 1$, oriented away from the z axis. Evaluate the following surface integrals

   (a)
   $$\iint_S x^9 y^{10} z^{11} \, dS$$

   (b)
   $$\iint_S z^2 \, dS$$

   (c)
   $$\iint_S (x^2 + y^2, x^2 + y^2, xyz) \cdot dS$$

   (d)
   $$\iint_S (x + y + z^2, \cos \sin(xz), 0) \cdot dS$$

5. Let
   $$F = \frac{(x, y, z)}{(x^2 + y^2 + z^2)^{3/2}}.$$

   Use the fact that $\nabla \cdot F = 0$ when $(x, y, z) \neq (0, 0, 0)$ to find the flux of F through the surface given by $x^2 + y^2 = 2 + z^2$, $-1 \leq z \leq 1$.