Homework 10

Due April 15th by 12:30pm by email to kdatchev@purdue.edu. Justify your answers. Please let me know if you have a question or find a mistake.

1. Let S be the surface parametrized by

$$x(u, v) = u \cos v, \qquad y(u, v) = u \sin v, \qquad z(u, v) = 3v,$$

for $0 \leq u \leq 4$ and $0 \leq v \leq \pi$. This is a variant of the helicoid example from the April 3rd lecture, and from several places in the book (see the index.)

(a) Find
$$\iint_S y \, dS.$$

(b) Find
$$\iint_S x \cos^3(1 + x^2 + y^2) \, dS.$$

2. Let $F = (x, x)$ and let c be the graph of $y = x^2$ from $(0, 0)$ to $(1, 1)$.

(a) Sketch c and the part of F on c.

(b) Find the line integral of F along c.

(c) Find the flux of F across c, oriented down and to the right.

3. Find
$$\iint_S (y, -x, z) \cdot dS,$$

where S is given by:

(a) The surface from problem 1 above, oriented upward.
  (*Hint:* $\cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha$)

(b) $x^2 + y^2 + z^2 = 4$, $x \geq 0$, $z \geq 0$, oriented away from the origin,