MA 510 second midterm review problems
Hopefully final version as of March 27th.

The second midterm will be a take-home exam assigned Wednesday, April 1st and due Thursday, April 2nd. It will be comparable in length to the first midterm. It will cover all the material from the classes and homework up to that date since the first midterm. That material approximately corresponds to chapter 5, chapter 6 (especially section 6.2), section 7.1, section 7.2, and section 8.1 of the book. While taking the exam, you may look up any information you need, but you may not use any calculating devices or programs or discuss the problems with anyone but me. Most of the problems on the exam will be closely based on ones from the list below. For each problem, you must justify your answers. Please let me know if you have a question or find a mistake.

1. Evaluate
$$\int_C \tan(\cos(\sin x^{x+y}))dx + x^2y^2dy,$$
where C is the line segment from $(2,2)$ to $(2,-2)$.

2. Which of the following vector fields can be written as ∇f for some function $f\colon \mathbb{R}^2 \to \mathbb{R}$? If the answer is yes, find such a function f.

   (a) $(3+2xy, x^2-3y^2)$,

   (b) $(2x\cos y - y\cos x, -x^2\sin y - \sin x)$,

   (c) $(ye^x + \sin y, e^x + x\cos y)$

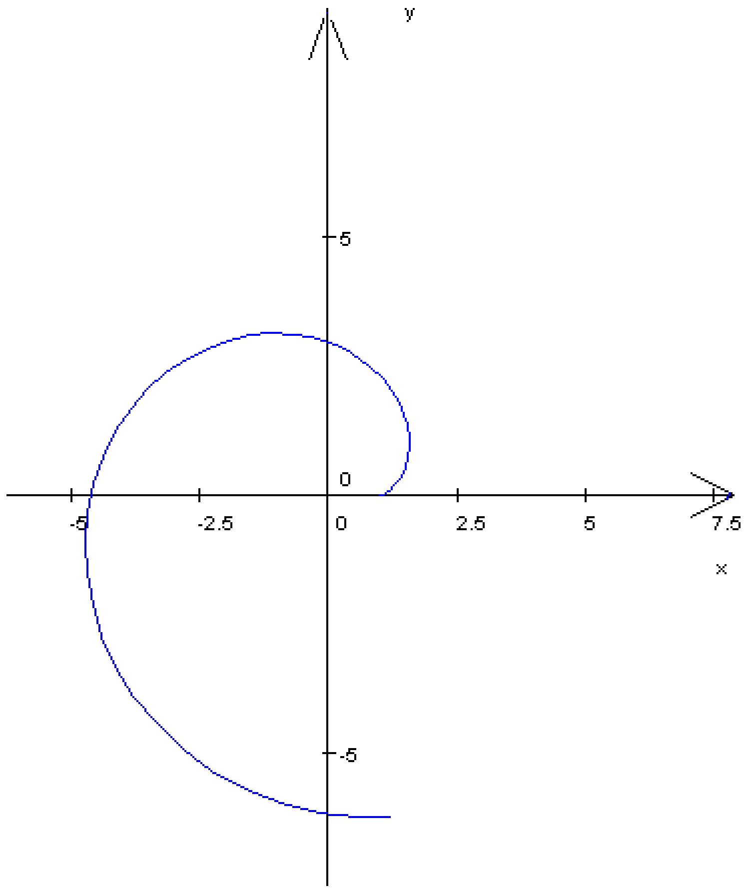
3. Evaluate
$$\int_C \sqrt{e^{\cos^3(x+y)}}(dx+dy),$$
where C is the circular arc beginning at $(0,0)$, passing through $(2,8)$, and ending at $(1,-1)$. Evaluate the same integral where C is a curve beginning at $(2\pi,-\pi)$ and ending at $(-\pi, 2\pi)$

4. Evaluate
$$\int_C 2xe^{x^2}\cos\sqrt{y}dx - \frac{e^{x^2}\sin\sqrt{y}}{2\sqrt{y}}dy,$$
where C is the circular arc beginning at $(0,\pi^2)$, passing through $(1,5)$, and ending at $(0,\pi^2/4)$.

5. Evaluate
$$\iint_D \cos(4x^2+9y^2)dxdy,$$

where D is the region given by $2 \leq 4x^2 + 9y^2 \leq 3$, $x \geq 0$, and $y \leq 0$.

6. A string of length 2π starts out wound around a spool given by the unit circle in $\mathbb{R}^2$, beginning and ending at $(1, 0)$. The top end of the string is unwound, while being kept taut, until it reaches the point $(1, -2\pi)$; the curve C traced by this end is called an *involute* of the circle. Here is a picture of C:



   (a) Find the length of C.

   (b) Find the area of the region bounded by C and the straight line segment joining its endpoints.

   Hint: C is parametrized by $x(t) = \cos t + t \sin t$ and $y(t) = \sin t - t \cos t$.

7. Use the change of variables $x = u^2$ and $y = v^3$ to find the area of the region where $x \geq 0$, $y \geq 0$, and $x^{1/2} + y^{1/3} \leq 1$.

8. Evaluate

$$\iiint_W z\,dx\,dy\,dz,$$

where W is the region inside the cone whose base is given by $x^2 + y^2 \leq 9$ in the plane

$z = 0$, and which has vertex given by $x = y = 0$ and $z = -5$.[1]

9. Evaluate $\int_C (y^3 - 2y)dx - x^3 dy$, where C is the circle $x^2 + y^2 = 4$, oriented clockwise.

10. Use Leibniz' rule for differentiating under the integral (see `https://www.math.purdue.edu/~kdatchev/510/leibniz.pdf`) to simplify

$$\frac{d}{dt}\int_{t^2}^{t^3} \frac{\cos(ty^2)}{y}\,dy.$$

11. Let W be the region given by $\sqrt{x^2 + y^2} \leq z$, $x^2+y^2+z^2 \leq 4$, and $x^2+y^2+(z-\frac{1}{2})^2 \geq \frac{1}{4}$. Use an integration in spherical coordinates to find the volume of W.

[1]This is not part of the problem, but we can use this to find the center of mass of W. If we multiply the result by $-g\mu$, where g is the acceleration of gravity and μ is the mass density of the ground, then this represents work done digging a hole in the shape of the region W.