Homework 1

Due August 31st in class or by 1:50 pm in MATH 602.

1. Let
$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$
 and $B = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

Which of the following expressions are defined and which are not? Evaluate the ones which are defined.

- (a) A^T
- (b) AB
- (c) $A^T + 2B$
- (d) 2A + B
- (e) $A^T B$
- (f) $B^T A$ (*Hint:* Use part (e)).
- 2. Consider the linear system

$$x_1 + 2x_2 + 3x_3 = 6$$

$$4x_1 + 5x_2 + 6x_3 = 15$$

$$7x_1 + 8x_2 + 9x_3 = 24$$

$$12x_1 + 15x_2 + 18x_3 = c,$$

where c is a real number,

- (a) Write down the associated augmented matrix and use Gaussian elimination to put it in row echelon form.
- (b) For which value or values of c does the system have at least one solution?
- (c) Use your answer to parts (a) and (b) to write down the general solution to the system, for those values of c for which it exists. Identify the bound and free variables. If there are any free variables, they should come after the bound variables in the ordering x_1, x_2, x_3 .

3. Consider the matrix

$$A = \begin{bmatrix} 3 & 5 & 2 & 3 \\ 0 & -2 & 1 & 3 \\ 1 & 3 & 0 & -1 \end{bmatrix}$$

- (a) Find a basis for the nullspace of A.
- (b) Find a basis for the row space of A.
- (c) Find a basis for the column space of A.
- (d) What is the rank of A?
- (e) What is the nullity of A?
- 4. Consider the vectors

$$\mathbf{u} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} 7 & 8 & c \end{bmatrix},$$

where c is a real number.

- (a) For which values of c are $\mathbf{u}, \mathbf{v}, \mathbf{w}$ linearly dependent and for which values are they linearly independent?
- (b) Find a basis for the span of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ for each value of c.