MA 527
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Fall 2017

Homework 10

Due November 30th in class or by 1:50 pm in MATH 602.

This homework covers sections 12.6 and 12.9.

1. Use the method of separation of variables for the partial differential equation
$$2\partial_x^2 u(x,y,t) + 3\partial_y^2 u(x,y,t) + \partial_t u(x,y,t) = 0$$
to derive three ordinary differential equations, involving two constants of separation. You do not need to solve the resulting ordinary differential equations.

2. In class we derived the formulas
$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-n^2 t} \sin nx$$
for the solution to
$$\partial_t u(x,t) = \partial_x^2 u(x,t), \qquad u(0,t) = u(\pi,t) = 0,$$
and
$$u(x,t) = \sum_{n=0}^{\infty} A_n e^{-n^2 t} \cos nx$$
for the solution to
$$\partial_t u(x,t) = \partial_x^2 u(x,t), \qquad \partial_x u(0,t) = \partial_x u(\pi,t) = 0.$$
(We also talked about how to write the coefficients A_n, B_n in terms of given initial conditions, but you can ignore that issue for this problem.) Derive a corresponding formula for
$$\partial_t u(x,t) = \partial_x^2 u(x,t), \qquad u(0,t) = \partial_x u(\pi,t) = 0.$$

3. (a) Let $u(x,t)$ be the solution to

$$\partial_t u(x,t) = \partial_x^2 u(x,t), \qquad u(0,t) = u(\pi,t) = 0, \qquad u(x,0) = f(x),$$

where

$$f(x) = \begin{cases} -1, & 0 \le x \le \pi/2, \\ 1, & \pi/2 < x < \pi. \end{cases}$$

Find the first nonzero term of the Fourier series for $u(x,t)$.

(b) Let $u(x,t)$ be the solution to

$$\partial_t u(x,t) = \partial_x^2 u(x,t), \qquad \partial_x u(0,t) = \partial_x u(\pi,t) = 0, \qquad u(x,0) = f(x),$$

where

$$f(x) = \begin{cases} -1, & 0 \le x \le \pi/2, \\ 1, & \pi/2 < x < \pi. \end{cases}$$

Find the first nonzero term of the Fourier series for $u(x,t)$.

(c) In which case is the convergence to equilibrium faster?

Aside: You may find it interesting to think about physical interpretations of this difference in rate of convergence to equilibrium, and to try to come up with examples of $f(x)$ such that the situation is reversed, but this is not part of the homework.