MA 527 Kiril Datchev Fall 2017

Homework 4 revised

Due September 28th in class or by 1:50 pm in MATH 602.

1. Consider the system

$$y_1' = y_1 + 4y_2, y_2' = y_1 + y_2.$$

- (a) Find the general solution.
- (b) Find the solution such that $y_1(0) = 1$ and $y_2(0) = 2$.
- 2. Consider the system

$$y_1' = -4y_1 - 3y_2, y_2' = -y_1 - 2y_2.$$

- (a) Sketch a phase portrait. Label all equilibrium and straight line solutions.
- (b) Decide which of the following terms apply to the description of the origin: improper node, proper node, saddle point, center, spiral, stable, unstable, attractive.
- 3. Choose one of the following:
 - (a) Find the general solution to the system

$$y'_1 = y_1 - 4y_2 + 2,$$

 $y'_2 = y_1 - 3y_2 + 1.$

(b) Find the general solution to the system

$$y'_1 = 6y_1 - 4y_2 + 2,$$

 $y'_2 = y_1 + y_2 + 1.$

Hint: Option (a) is from the original version of the assignment so you can submit that if you already did it, but (b) is meant to be easier.

4. Consider the nonlinear system

$$y'_1 = -y_2,$$

 $y'_2 = y_1 - \frac{1}{2}y_1^2.$

- (a) Find the critical points of the system.
- (b) Find the linearized system at each critical point. Decide which of the following terms apply to the description of the critical point of each linearized system: improper node, proper node, degenerate node, saddle point, center, spiral, stable, unstable, attractive.
- (c) What can you conclude about the critical points of the nonlinear system from the results of part (b)?