

Homework 9

Due November 9th in class or by 1:50 pm in MATH 602.

This homework covers section 12.3.

1. Use the method of separation of variables for the partial differential equation

$$2\partial_x^2 u(x, y) + 3\partial_y^2 u(x, y) = 0$$

to derive a pair of ordinary differential equations involving a constant of separation. You do not need to solve the resulting ordinary differential equations.

2. When $0 \leq x \leq 10$, define $f(x)$ by

$$f(x) = \begin{cases} 0, & 0 \leq x \leq 4 \\ 1, & 4 \leq x \leq 6 \\ 0, & 6 \leq x \leq 10. \end{cases}$$

- (a) Sketch the graph of $f(x)$ for $0 \leq x \leq 10$.
 - (b) Let $u(x, t) = \frac{1}{2}(f^*(x - t) + f^*(x + t))$, where f^* is the odd extension of f with period 20. Sketch the graphs of $u(x, 0)$, $u(x, .5)$, $u(x, 2)$, $u(x, 4.5)$, $u(x, 5)$, $u(x, 8)$, and $u(x, 10)$, for $0 \leq x \leq 10$.
3. Find the eigenvalues and eigenvectors of the Sturm–Liouville problem

$$y''(x) + \lambda y(x) = 0, \quad y(0) = y'(\pi) = 0.$$