MA 223 - HOW TO EVALUATE A LIMIT

Step 1 in evaluating a limit, $\lim\limits_{x \to ?} f(x)$, is determining whether the limit is infinite ($x \to \infty$ or $-\infty$) or finite ($x \to c$ for any finite constant c, like $c = -3$).

FINITE LIMITS Evaluating $\lim\limits_{x \to c} \dfrac{p(x)}{q(x)}$ ...

1. Check if $q(c) = 0$, i.e. check if plugging in c for x causes a 0 in the denominator.

   (a) If $q(c) \neq 0$ (meaning if the denominator is not zero when you plug in c), then plug in c to evaluate! $\lim\limits_{x \to c} \dfrac{p(x)}{q(x)} = \dfrac{p(c)}{q(c)}$ is the answer.

   (b) If $q(c) = 0$ (meaning if the denominator is zero when you plug in c), then more work must be done.

2. Factor the numerator and denominator.

3. Cancel terms. Now can you plug in c for x without getting a 0 in the denominator?

   The denominator keeps being 0, what do I do? This can happen in situations like this: $\lim\limits_{x \to -1} \dfrac{1}{x+1}$ or this $\lim\limits_{x \to 5} \dfrac{x-4}{5-x}$.

   When you've gone through the above steps but plugging in $x = c$ still causes division by zero, there are three possibilities left:

   (1) The limit does not exist, DNE; (2) the limit is ∞; or (3) the limit is $-\infty$.

   Look at the one-sided limit PDF to figure out this case!

INFINITE LIMITS Evaluating $\lim\limits_{x \to \infty} \dfrac{p(x)}{q(x)}$ or $\lim\limits_{x \to -\infty} \dfrac{p(x)}{q(x)}$

Look at the degrees of the numerator (NUM) and denominator (DENOM).

1. If they have **equal degrees**, then the limit equals the ratio of the leading coefficients.

2. If degree of **DENOM is bigger**, then the limit is 0.

3. If degree of **NUM is bigger**, then the answer is either ∞ or $-\infty$. Determining which requires more work:

    (a) Divide every term in the numerator and denominator by the highest power of x that appears in the denominator.

    (b) If $x \to \infty$, plug in ∞ for x. If $x \to -\infty$, plug in $-\infty$ for x. Figure out whether the numerator is positive or negative, and figure out whether the denominator is positive or negative. That will tell you whether the final answer is positive or negative ∞.