MA 223 - ONE-SIDED LIMITS

What do you do when Evaluating $\lim_{x \to c} \frac{p(x)}{q(x)}$, but plugging in c for x makes the denominator 0 no matter what you do?

This happens in situations like this: $\lim_{x \to -1} \frac{1}{x+1}$ or this $\lim_{x \to 5} \frac{x-4}{5-x}$.

The possible results are

(1) The limit does not exist, DNE; (2) the limit is ∞; or (3) the limit is $-\infty$.

To determine which, you look at the LEFT limit, $L = \lim_{x \to c^-} f(x)$ and the RIGHT limit $R = \lim_{x \to c^+} f(x)$:

If $L = R$, then the actual limit exists and is equal to L and R, i.e. $\lim_{x \to c} f(x) = \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$. The answer will be ∞ or $-\infty$, and you have to figure out which using the procedure below.

If $L \neq R$ then the limit does not exist, and the answer is DNE. This happens when one side is $-\infty$ and the other side is ∞.

LEFT LIMIT When evaluating $L = \lim_{x \to c^-} f(x)$, the key is that since $x \to c^-$, x is "to the left" of c, meaning $x < c$, meaning $(x - c) < 0$, so $(x - c)$ is negative. Use this to figure out whether the denominator is positive or negative. Then figure out whether the numerator goes to positive or negative infinity. Finally, figure out if L is ∞ or $-\infty$.

RIGHT LIMIT When evaluating $R = \lim_{x \to c^+} f(x)$, the key is that since $x \to c^+$, x is "to the right" of c, meaning $x > c$, meaning $(x - c) > 0$, so $(x - c)$ is positive. Use this to figure out whether the denominator is positive or negative. Then figure out whether the numerator goes to positive or negative infinity. Finally, figure out if R is ∞ or $-\infty$.