

MA 224 - Quiz 11

SOLUTIONS

NOTE: My solution for a given problem is not necessarily the only correct way to do that problem; I accept any method of solving the problems that follows the instructions and uses material taught in this course. I give a lot of detail to make sure everything is clear—I don't expect you to give quite this much detail.

1. (4 points) Give the general solution to this differential equation (“general” means you should have “+C” at the end). You should simplify your final answer so it's in the form “ $y = [\text{stuff with } x\text{'s in it}]$ ”. Show your work for partial credit!

$$\frac{dy}{dx} = \frac{4}{x\sqrt{y}}$$

Use the standard method of solving: separation of variables!

$$\begin{aligned} \frac{dy}{dx} &= \frac{4}{x\sqrt{y}} \\ \sqrt{y} \, dy &= \frac{4}{x} \, dx && \text{separate} \\ y^{1/2} \, dy &= 4x^{-1} \, dx && \text{simplify a little} \\ \int y^{1/2} \, dy &= 4 \int x^{-1} \, dx && \text{integrate} \\ \frac{y^{3/2}}{3/2} &= 4 \ln|x| + C_1 \\ y^{3/2} &= 6 \ln|x| + C_2 && \text{simplify} \\ (y^{3/2})^{2/3} &= (6 \ln|x| + C_2)^{2/3} && \text{undo the “3/2” power} \\ y &= (6 \ln|x| + C_2)^{2/3} && \text{final answer} \end{aligned}$$

2. (6 points) Solve this initial value problem if $s = 1$ when $t = 0$.

$$\frac{ds}{dt} = s^4 t^2$$

Since we're solving an initial value problem, we have two basic steps: first find the general solution using separation of variables. Then, plug in the initial value information to find the constant C .

$$\begin{aligned} \frac{ds}{dt} &= s^4 t^2 \\ s^{-4} ds &= t^2 dt && \text{separate} \\ \int s^{-4} ds &= \int t^2 dt && \text{integrate} \\ \frac{s^{-3}}{-3} &= \frac{t^3}{3} + C_1 \\ s^{-3} &= -t^3 + C_2 && \text{multiply by -3} \\ \frac{1}{s^3} &= -t^3 + C_2 \\ s^3 &= \frac{1}{-t^3 + C_2} \\ s &= \left(\frac{1}{-t^3 + C_2} \right)^{1/3} \end{aligned}$$

Now we have the general solution. If we plug in the initial value information, $s = 1$ when $t = 0$, we get

$$\begin{aligned} 1 &= \left(\frac{1}{0 + C_2} \right)^{1/3} \\ 1^3 &= \frac{1}{C_2} \\ 1 &= \frac{1}{C_2} \\ C_2 &= 1 \end{aligned}$$

So the final answer is

$$s = \left(\frac{1}{-t^3 + 1} \right)^{1/3}$$