MA 224 - Quiz 2

SOLUTIONS

NOTE: My solution for a given problem is not necessarily the only correct way to do that problem; I accept any method of solving the problems that follows the instructions and uses material taught in this course. I give a lot of detail to make sure everything is clear—I don't expect you to give quite this much detail.

1. (5 pts)

$$\int \frac{7w^2}{5w^3 - 20} dw$$

There is no basic integral rule for this, so some manipulation is needed. Your first instinct should be (1) simplify by factoring out the constant, then (2) seeing if any u-substitution will work readily.

$$\int \frac{7w^2}{5w^3 - 20} dw = \frac{7}{5} \int \frac{w^2}{w^3 - 4} dw$$

The only likely candidate for a u-substitution is the denominator: letting $u = w^3 - 4$ seems potentially helpful, since it would make the denominator easier to handle, and especially since the derivative of $w^3 - 4$ is $3w^2$—which is very near something that appears elsewhere in the integrand. Remember, it's key to choose u so that its derivative is also present in the integrand.

So let's try $u = w^3 - 4$, $du = 3w^2 dw$. Solving for dw gives $dw = \frac{du}{3w^2}$. Now we plug this in:

$$\begin{aligned}
\frac{7}{5} \int \frac{w^2}{w^3 - 4} dw &= \frac{7}{5} \int \frac{w^2}{u} dw && \text{plugging in } u \\
&= \frac{7}{5} \int \frac{w^2}{u} \frac{du}{3w^2} && \text{plugging in for } dw \\
&= \frac{7}{5} \cdot \frac{1}{3} \int \frac{w^2}{uw^2} du && \text{simplifying} \\
&= \frac{7}{15} \int \frac{du}{u} && \text{simplifying more} \\
&= \frac{7}{15} \cdot \ln |u| + C && \text{at last using a basic rule} \\
&= \frac{7}{15} \cdot \ln |w^3 - 4| + C && \text{return to original variable, } w
\end{aligned}$$

2. (5 pts)

$$\int \left(5t^4 e^{t^5 + t} + e^{t^5 + t} \right) dt$$

Once again, no basic rule works. Even worse, there are two seperate *different* pieces for which we have no rule: $5t^4e^{t^5+t}$ and e^{t^5+t}. If we try breaking this integral into two pieces: $\int 5t^4e^{t^5+t}dt + \int e^{t^5+t}dt$, then quickly we see that one piece is impossible: $\int e^{t^5+t}dt$. So we need to find some other way.

Instead, try factoring first: $\int \left(5t^4e^{t^5+t} + e^{t^5+t}\right) dt = \int e^{t^5+t}\left(5t^4+1\right) dt$. Still no basic rule applies, but at least now there is only one problematic piece, e^{t^5+t}, instead of two. Since e^u is something for which we have a basic rule, a u-substitution could make this manageable if we can get all the t terms to cancel properly, so the next step is trying out a u-substitution:

let $u = t^5 + t$, which gives $du = (5t^4 + 1)dt$. Solving for dt gives $dt = \frac{du}{(5t^4+1)}$. Plugging these in:

$$\begin{aligned}
\int e^{t^5+t}\left(5t^4+1\right) dt &= \int e^u \left(5t^4+1\right) \frac{du}{(5t^4+1)} && \text{substituting} \\
&= \int e^u du && \text{cancelling} \\
&= e^u + C && \text{using the basic rule} \\
&= e^{t^5+t} + C && \text{returning to original variable}
\end{aligned}$$