MA 224 - Quiz 3

SOLUTIONS

NOTE: My solution for a given problem is not necessarily the only correct way to do that problem; I accept any method of solving the problems that follows the instructions and uses material taught in this course. I give a lot of detail to make sure everything is clear—I don't expect you to give quite this much detail.

1. (2 pts) $\int \frac{7x}{(x-2)^3}dx$
   Let $u = x - 2$, so $du = dx$. Then use $x = u + 2$ to complete the u-substitution and get to $\int \frac{7(u+2)}{u^3}du = 7\int (u^{-2} + 2u^{-3})\, du$.

   Final answer: $-\frac{7}{x-2} - \frac{7}{(x-2)^2} + C$

2. (2 pts) $\int \frac{\sqrt[5]{2\ln(x)}}{6x}dx$
   Factor the 2 outside of the fifth root: $\sqrt[5]{2\ln(x)} = 2^{1/5} \cdot (\ln(x))^{1/5}$. Then factor the constant in the numerator and denominator outside the integral: $\frac{2^{1/5}}{6}\int \frac{(\ln(x))^{1/5}}{x}dx$. Now let $u = \ln(x)$, so $du = \frac{dx}{x}$. Substituting and simplifying will give $\frac{2^{1/5}}{6}\int u^{1/5}du$. Then integrating and returning to our original variable gives us:

   Final answer: $\frac{5\cdot 2^{1/5}}{36}(\ln(x))^{6/5} + C$

3. (2 pts) $\int_0^1 \left(e^{2x^3+2x} + 3x^2 e^{2x^3+2x}\right) dx$
   First factor out e^{2x^3+2x} to get $\int_0^1 e^{2x^3+2x}\left(1 + 3x^2\right) dx$. Then let $u = 2x^3+2x$ so $du = (6x^2+2)dx = 2(3x^2+1)dx$. Substituting in gives $\int_{x=0}^{x=1} e^u \left(1+3x^2\right)\frac{du}{2(3x^2+1)} = \frac{1}{2}\int_{x=0}^{x=1} e^u du$. Finally, we can solve by finding an antiderivative, then evaluating $(F(x)|_0^1$ once we have our variable matching our limits of integration.

   Final answer: $\frac{1}{2}(e^4 - 1)$

4. (2 pts) $\int_3^4 \left(x\sqrt{2x-6} + \sqrt{8x-24}\right) dx = \int_3^4 \left(x\sqrt{2x-6} + \sqrt{4(2x-6)}\right) dx$
   $= \int_3^4 \left(x\sqrt{2x-6} + 2\sqrt{2x-6}\right) dx$. Let $u = 2x - 6$. Then $du = 2dx$, but you have to make the second substitution $x = \frac{1}{2}(u+6)$ to get $\int_{x=3}^{x=4}\left(\frac{1}{2}(u+6)\sqrt{u} + 2\sqrt{u}\right)\frac{du}{2}$. Distributing and simplifying gives $\int_{x=3}^{x=4}\left(\frac{1}{4}u^{3/2} + \frac{5}{2}u^{1/2}\right) du$. Taking the antideravtive and replacing u with xs gives $\left(\frac{1}{10}(2x-6)^{5/2} + \frac{5}{3}(2x-6)^{3/2}\right|_3^4$ which gives

Final answer: $\frac{2^{5/2}}{10} + \frac{5}{3} \cdot (2^{3/2}) = \frac{56\sqrt{2}}{15}$

5. (2 pts) $\int_{-1}^{1} 8(x+8)(x+1)^6 dx$

Let $u = x + 1$. Then $du = dx$. Substituting gives $8\int_{x=-1}^{x=1} (x + 8)u^6 du$. We need to make another substitution to remove the last x. Using the equation $u = x + 1$ we get $x = u - 1$, and substituting gives $8\int_{x=-1}^{x=1} (u - 1 + 8)u^6 du = 8\int_{x=-1}^{x=1} \left(u^7 + 7u^6\right) du = 8\left(\frac{1}{8}u^8 + u^7\right|_{x=-1}^{x=1} = \left((x+1)^8 + 8(x+1)^7\right|_{-1}^{1}$ and evaluating this gives

Final answer: 1280