MA 224 - Quiz 5

SOLUTIONS

NOTE: My solution for a given problem is not necessarily the only correct way to do that problem; I accept any method of solving the problems that follows the instructions and uses material taught in this course. I give a lot of detail to make sure everything is clear—I don't expect you to give quite this much detail.

1. (5 pts)

$$\int (1+2t)e^{-t}\,\mathrm{d}t$$

Since this quiz is on Integration by Parts, you knew not to try u-substitution. But if this was an exam problem (and you weren't told ahead of time to use Integration by Parts), you could figure out that Int. by Pts. is the right method by noting that this is 2 functions times each other, not one function stuffed inside of another.

So we have to pick u and $\mathrm{d}v$. Since both $(1+2t)$ and e^{-t} are easy to integrate, either one of them could be $\mathrm{d}v$. THat means we have to be careful when we pick u: we have to make sure the derivative is *simpler* than u is. Letting $u = e^{-t}$ makes $\mathrm{d}u = -e^{-t}$, which is not any simpler. But letting $u = (1+2t)$ makes $\mathrm{d}u = 2\,\mathrm{d}t$, which is simpler! So we have $u = (1+2t)$ and $\mathrm{d}v = e^{-t}$.

Now we find the other pieces: $\mathrm{d}u = 2\,\mathrm{d}t$, and integrating $\mathrm{d}v = e^{-t}$ gives $v = -e^{-t}$.

At last we plug the pieces into our formula and simplify:

$$\begin{aligned}
\int (1+2t)e^{-t}\,\mathrm{d}t &= uv - \int v\,\mathrm{d}u \\
&= (1+2t)(-e^{-t}) - \int (-e^{-t})2\,\mathrm{d}t && \text{plugging in} \\
&= -(1+2t)e^{-t} + 2\int e^{-t}\,\mathrm{d}t \\
&= -(1+2t)e^{-t} - 2e^{-t} + C \\
&= -e^{-t}\,(1+2t+2) + C && \text{factoring out } -e^{-t}
\end{aligned}$$

giving the final answer $-e^{-t}\,(3+2t)+C$

2. (5 pts)

$$\int 4\frac{\ln(x)}{\sqrt{x}}\,\mathrm{d}x$$

Again you could figure out that Int. by Pts. is the right method by noting that this is 2 functions multiplied together, not one function stuffed inside of another.

To pick u and $\mathrm{d}v$, first rewrite the problem as $\int 4\ln(x)x^{-1/2}\,\mathrm{d}x$. Since $\ln(x)$ is one of the terms, and we can't integrate $\mathrm{d}v = \ln(x)$, it is a bad idea to let $\mathrm{d}v = \ln(x)$. So instead, let $u = \ln(x)$! Then let $\mathrm{d}v$ be equal to everything else: $\mathrm{d}v = 4x^{-1/2}\,\mathrm{d}x$.

Now we find the other pieces: the derivative of $u = \ln(x)$ is $\mathrm{d}u = x^{-1}\,\mathrm{d}x$, and integrating $\mathrm{d}v = 4x^{-1/2}\,\mathrm{d}x$ gives $v = 4x^{1/2}/(1/2) = 8x^{1/2}$.

At last we plug the pieces into our formula and simplify:

$$\begin{aligned}
\int 4\frac{\ln(x)}{\sqrt{x}}\,\mathrm{d}x &= uv - \int v\,\mathrm{d}u \\
&= \ln(x)8x^{1/2} - \int 8x^{1/2}x^{-1}\,\mathrm{d}x && \text{plugging in} \\
&= \ln(x)8x^{1/2} - 8\int x^{-1/2}\,\mathrm{d}x && \text{simplifying } x^{1/2}x^{-1} = x^{-1/2} \\
&= \ln(x)8x^{1/2} - 8x^{1/2}/(1/2) + C && \text{integrating using power rule} \\
&= \ln(x)8x^{1/2} - 16x^{1/2} + C && \text{simplifying}
\end{aligned}$$

and factoring gives the final answer $8x^{1/2}\left(\ln(x) - 2\right) + C$